Forecasting Portfolio Balance Using Return Mean, Standard Deviation and Spending

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Abstract

This paper develops an integrated formula using return mean, standard deviation and spending to forecast the ending balance of a portfolio. The forecasted ending balances were robust when tested over a variety of time periods, spending percentages, and varying how the spending was calculated.

This formula is useful for a variety of stakeholders – for government regulators to see how a change in required spending percentages would affect the long-term viability of institutions, for those institutions in understanding how the standard deviation (as a proxy for volatility) affects the portfolio balance, and for investment committees in understanding the trade-off between return and volatility and the resultant effect on the portfolio, among others.

Keywords: portfolio management, endowment, foundation, forecasting, portfolio balance, volatility, investment, return, standard deviation

1. Introduction

There are three principal metrics that will determine the ending balance of an endowment portfolio:

1. Return
2. Standard deviation of return
3. Rate of spending

Currently there appears to be a lack of research clearly demonstrating the specific interaction of those three variables, although all certainly are recognized as being critical to the management of an endowment portfolio. Conceptually there is a fourth variable that should be considered, namely capital additions (new money) that could flow into the endowment. For a college, university, museum, or pension, potential inflows are an important element that should be taken into account. For a private foundation or retiree, however, the omission of inflows is a reasonable assumption.

The motivation for this research was a problem devised as "food for thought" for the institutional investor community (Haber (2014a)). The problem was as follows:

> *Let's say you had an endowment with a balance of $500,000,000. While on vacation, you find a lamp and rub it, and out pops a genie who grants you some wishes. With your first wish, you learn that over the next ten years you will experience five years of annual returns of +5% and five years of annual returns of -5%. With your second wish, you can select the order in which the returns happen (five years of +5% followed by five years of -5%, or five years of -5% followed by five years of +5%, alternating years of positive and negative or negative and positive, and so on).*
>
> *Without using a spreadsheet, what do you choose?*

The answer is that the order of the returns does not matter. You will always wind up with the same ending portfolio balance. The problem was further expanded by introducing spending into the mix (Haber (2014b)):

> *For simplicity, let both the annual return and the annual spending be apportioned evenly throughout the year, and assume that the annual return and the annual spending occur at the same time of year (as opposed to spending happening at the beginning of the year and return at the end of the year, for instance).*
>
> *Using a single spending factor for the 10 years, which of the following arrangements will produce the highest ending portfolio balance?:*

1. *Positive returns first, negative returns follow*
2. *Negative returns first, positive returns follow*
3. *Alternating positive and negative, starting with positive*
4. *Alternating negative and positive, starting with negative*

The answer, again, is that is does not matter. You will always arrive at the same ending endowment balance. This basic situation clearly is not often repeated in practice, given that returns can be highly volatile on a month-to-month basis. If spending is based on a past balance (such as prior month or prior year), then it would seem that there is the potential to be spending more (based on a higher previous endowment balance) than is warranted (based on a lower current endowment balance), and vice versa.

This paper incorporates spending using two mechanisms:

1. Based on 1/12 of the previous month's endowment balance
2. Based on 1/12 of the endowment balance at the previous year-end

There certainly are other common ways to incorporate a spending pattern (rolling 12-quarter periods, for example), so there is room for future research with a greater variety of spending rules.

The specific focus of this research is to develop a single, robust equation to predict a portfolio balance, utilizing a set of relevant variables. Such an equation would be extremely useful in shedding light on the interaction of the critical elements in the behavior of a portfolio, and thus provide insight for portfolio management. For example, consider an endowment whose spending is based on prior balances. One might well speculate that volatile returns could lead to spending that ultimately cannot be sustained, resulting in a negative spiral for the endowment. The type of equation presented in this study can help provide evidence as to whether such fears are indeed legitimate.

2. Process

Rather than using manufactured returns, we used the actual S&P 500 monthly returns for the 10 year period from May 1, 2005 through April 30, 2015 (see Figure 1):

N	Month End	S&P 500	S&P 500 Monthly Return
0	05/31/05	1191.50	
1	06/30/05	1191.33	-0.0143
2	07/29/05	1234.18	3.5968
3	08/31/05	1220.33	-1.1222
4	09/30/05	1228.81	0.6949
5	10/31/05	1207.01	-1.7741
6	11/30/05	1249.48	3.5186
7	12/30/05	1248.29	-0.0952
8	01/31/06	1280.08	2.5467
9	02/28/06	1280.66	0.0453
10	03/31/06	1294.83	1.1065
11	04/28/06	1310.61	1.2187
12	05/31/06	1270.09	-3.0917
13	06/30/06	1270.20	0.0087
14	07/31/06	1276.66	0.5086
15	08/31/06	1303.82	2.1274
16	09/29/06	1335.85	2.4566
17	10/31/06	1377.94	3.1508
18	11/30/06	1400.63	1.6467
19	12/29/06	1418.30	1.2616
20	01/31/07	1438.24	1.4059
21	02/28/07	1406.82	-2.1846
22	03/30/07	1420.86	0.9980
23	04/30/07	1482.37	4.3291
24	05/31/07	1530.62	3.2549
25	06/29/07	1503.35	-1.7816

26	07/31/07	1455.27	-3.1982
27	08/31/07	1473.99	1.2864
28	09/28/07	1526.75	3.5794
29	10/31/07	1549.38	1.4822
30	11/30/07	1481.14	-4.4043
31	12/31/07	1468.36	-0.8628
32	01/31/08	1378.55	-6.1163
33	02/28/08	1367.68	-0.7885
34	03/31/08	1322.70	-3.2888
35	04/30/08	1385.59	4.7547
36	05/30/08	1400.38	1.0674
37	06/30/08	1280.00	-8.5962
38	07/31/08	1267.38	-0.9859
39	08/29/08	1282.83	1.2191
40	09/30/08	1166.36	-9.0791
41	10/31/08	968.75	-16.9425
42	11/28/08	896.24	-7.4849
43	12/31/08	903.25	0.7822
44	01/30/09	825.88	-8.5657
45	02/27/09	735.09	-10.9931
46	03/31/09	797.87	8.5405
47	04/30/09	872.81	9.3925
48	05/29/09	919.14	5.3081
49	06/30/09	919.32	0.0196
50	07/31/09	987.48	7.4142
51	08/31/09	1020.62	3.3560
52	09/30/09	1057.08	3.5723
53	10/30/09	1036.19	-1.9762
54	11/30/09	1095.63	5.7364
55	12/31/09	1115.10	1.7771
56	01/29/10	1073.87	-3.6974
57	02/26/10	1104.49	2.8514
58	03/31/10	1169.43	5.8796
59	04/30/10	1186.69	1.4759
60	05/28/10	1089.41	-8.1976
61	06/30/10	1030.71	-5.3882
62	07/30/10	1101.60	6.8778
63	08/31/10	1049.33	-4.7449
64	09/30/10	1141.20	8.7551
65	10/29/10	1183.26	3.6856
66	11/30/10	1180.55	-0.2290
67	12/31/10	1257.64	6.5300
68	01/31/11	1286.12	2.2646
69	02/28/11	1327.22	3.1957
70	03/31/11	1325.83	-0.1047
71	04/29/11	1363.61	2.8495
72	05/31/11	1345.20	-1.3501
73	06/30/11	1320.64	-1.8258
74	07/29/11	1292.28	-2.1474
75	08/31/11	1218.89	-5.6791
76	09/30/11	1131.42	-7.1762
77	10/31/11	1253.30	10.7723
78	11/30/11	1246.96	-0.5059
79	12/30/11	1257.60	0.8533
80	01/31/12	1312.41	4.3583

81	02/29/12	1365.68	4.0589
82	03/30/12	1408.47	3.1332
83	04/30/12	1397.91	-0.7497
84	05/31/12	1310.33	-6.2651
85	06/29/12	1362.16	3.9555
86	07/31/12	1379.32	1.2598
87	08/31/12	1406.58	1.9763
88	09/28/12	1440.67	2.4236
89	10/31/12	1412.16	-1.9789
90	11/30/12	1416.18	0.2847
91	12/31/12	1426.19	0.7068
92	01/31/13	1498.11	5.0428
93	02/28/13	1514.68	1.1061
94	03/28/13	1569.19	3.5988
95	04/30/13	1597.57	1.8086
96	05/31/13	1630.74	2.0763
97	06/28/13	1606.28	-1.4999
98	07/31/13	1685.73	4.9462
99	08/30/13	1632.97	-3.1298
100	09/30/13	1681.55	2.9749
101	10/31/13	1756.54	4.4596
102	11/29/13	1805.81	2.8049
103	12/31/13	1848.36	2.3563
104	01/31/14	1782.59	-3.5583
105	02/28/14	1859.45	4.3117
106	03/31/14	1872.34	0.6932
107	04/30/14	1883.95	0.6201
108	05/30/14	1923.57	2.1030
109	06/30/14	1960.23	1.9058
110	07/31/14	1930.67	-1.5080
111	08/29/14	2003.37	3.7655
112	09/30/14	1972.29	-1.5514
113	10/31/14	2018.05	2.3201
114	11/28/14	2067.56	2.4534
115	12/31/14	2058.90	-0.4189
116	01/30/15	1994.99	-3.1041
117	02/27/15	2104.50	5.4893
118	03/31/15	2067.89	-1.7396
119	04/30/15	2085.51	0.8521

Figure 1. Monthly S&P 500 returns

The arithmetic mean return during this period was 0.5639%, and the compound mean return was 0.4715%. The standard deviation of the actual monthly returns was 4.2677%. We utilized a starting endowment balance of $500,000,000 and four spending rates (0%, 2%, 5%, 8%), applied according to two schemes (one based on the prior month's ending endowment balance and one based on the prior year's ending endowment balance) (the tables follow the list below).

Figure 2 – 0% spending

Figure 3 – 2% spending, based on prior month's endowment balance

Figure 4 – 5% spending, based on prior month's endowment balance

Figure 5 – 8% spending, based on prior month's endowment balance

Figure 6 – 2% spending, based on prior year's endowment balance

Figure 7 – 5% spending, based on prior year's endowment balance

Figure 8 – 8% spending, based on prior year's endowment balance

Month End	Apply Monthly S&P 500 Returns
05/31/05	500,000,000
06/30/05	499,928,661
07/29/05	517,910,197
08/31/05	512,098,196
09/30/05	515,656,735
10/31/05	506,508,603
11/30/05	524,330,676
12/30/05	523,831,305
01/31/06	537,171,632
02/28/06	537,415,023
03/31/06	543,361,309
04/28/06	549,983,214
05/31/06	532,979,438
06/30/06	533,025,598
07/31/06	535,736,467
08/31/06	547,133,865
09/29/06	560,574,906
10/31/06	578,237,516
11/30/06	587,759,127
12/29/06	595,174,150
01/31/07	603,541,754
02/28/07	590,356,693
03/30/07	596,248,426
04/30/07	622,060,428
05/31/07	642,308,015
06/29/07	630,864,457
07/31/07	610,688,208
08/31/07	618,543,852
09/28/07	640,684,012
10/31/07	650,180,445
11/30/07	621,544,272
12/31/07	616,181,284
01/31/08	578,493,496
02/28/08	573,932,018
03/31/08	555,056,651
04/30/08	581,447,755
05/30/08	587,654,217
06/30/08	537,138,061
07/31/08	531,842,216
08/29/08	538,325,640
09/30/08	489,450,273
10/31/08	406,525,388
11/28/08	376,097,356
12/31/08	379,039,026
01/30/09	346,571,548
02/27/09	308,472,514
03/31/09	334,817,457
04/30/09	366,265,212

05/29/09	385,707,092
06/30/09	385,782,627
07/31/09	414,385,229
08/31/09	428,292,069
09/30/09	443,592,111
10/30/09	434,825,850
11/30/09	459,769,198
12/31/09	467,939,572
01/29/10	450,637,851
02/26/10	463,487,201
03/31/10	490,738,565
04/30/10	497,981,536
05/28/10	457,159,043
06/30/10	432,526,227
07/30/10	462,274,444
08/31/10	440,339,908
09/30/10	478,892,153
10/29/10	496,542,174
11/30/10	495,404,952
12/31/10	527,754,931
01/31/11	539,706,253
02/28/11	556,953,420
03/31/11	556,370,122
04/29/11	572,224,087
05/31/11	564,498,531
06/30/11	554,192,195
07/29/11	542,291,230
08/31/11	511,493,915
09/30/11	474,788,082
10/31/11	525,933,697
11/30/11	523,273,185
12/30/11	527,738,145
01/31/12	550,738,565
02/29/12	573,092,740
03/30/12	591,049,098
04/30/12	586,617,709
05/31/12	549,865,715
06/29/12	571,615,611
07/31/12	578,816,618
08/31/12	590,255,980
09/28/12	604,561,477
10/31/12	592,597,566
11/30/12	594,284,515
12/31/12	598,485,103
01/31/13	628,665,548
02/28/13	635,618,968
03/28/13	658,493,496
04/30/13	670,402,854
05/31/13	684,322,283
06/28/13	674,057,910
07/31/13	707,398,238
08/30/13	685,258,078
09/30/13	705,644,146
10/31/13	737,112,883
11/29/13	757,788,502

Month End	Balance
12/31/13	775,644,146
01/31/14	748,044,482
02/28/14	780,297,944
03/31/14	785,707,092
04/30/14	790,579,102
05/30/14	807,205,204
06/30/14	822,589,173
07/31/14	810,184,641
08/29/14	840,692,405
09/30/14	827,650,021
10/31/14	846,852,707
11/28/14	867,629,039
12/31/14	863,994,964
01/30/15	837,175,829
02/27/15	883,130,508
03/31/15	867,767,520
04/30/15	875,161,561

Figure 2. 0% Spending

Spend 2%/12 of Prior Month's Balance			
Month End	Spend	Return	Balance
05/31/05			500,000,000
06/30/05	833,333	-71,339	499,095,328
07/29/05	831,826	17,951,562	516,215,065
08/31/05	860,358	-5,792,979	509,561,728
09/30/05	849,270	3,540,914	512,253,372
10/31/05	853,756	-9,087,754	502,311,862
11/30/05	837,186	17,674,406	519,149,081
12/30/05	865,248	-494,436	517,789,397
01/31/06	862,982	13,186,459	530,112,874
02/28/06	883,521	240,192	529,469,545
03/31/06	882,449	5,858,373	534,445,468
04/28/06	890,742	6,513,248	540,067,974
05/31/06	900,113	-16,697,228	522,470,633
06/30/06	870,784	45,250	521,645,099
07/31/06	869,408	2,652,990	523,428,680
08/31/06	872,381	11,135,559	533,691,858
09/29/06	889,486	13,110,821	545,913,192
10/31/06	909,855	17,200,648	562,203,985
11/30/06	937,007	9,257,594	570,524,572
12/29/06	950,874	7,197,596	576,771,294
01/31/07	961,285	8,108,877	583,918,885
02/28/07	973,198	-12,756,377	570,189,310
03/30/07	950,316	5,690,464	574,929,458
04/30/07	958,216	24,889,089	598,860,331
05/31/07	998,101	19,492,442	617,354,672
06/29/07	1,028,924	-10,998,982	605,326,766
07/31/07	1,008,878	-19,359,504	584,958,384
08/31/07	974,931	7,524,666	591,508,119
09/28/07	985,847	21,172,442	611,694,715
10/31/07	1,019,491	9,066,744	619,741,967
11/30/07	1,032,903	-27,295,558	591,413,506
12/31/07	985,689	-5,103,005	585,324,812

01/31/08	975,541	-35,800,499	548,548,771
02/28/08	914,248	-4,325,360	543,309,163
03/31/08	905,515	-17,868,249	524,535,399
04/30/08	874,226	24,939,919	548,601,093
05/30/08	914,335	5,855,852	553,542,610
06/30/08	922,571	-47,583,841	505,036,198
07/31/08	841,727	-4,979,341	499,215,130
08/29/08	832,025	6,085,684	504,468,788
09/30/08	840,781	-45,801,454	457,826,552
10/31/08	763,044	-77,567,050	379,496,458
11/28/08	632,494	-28,404,943	350,459,021
12/31/08	584,098	2,741,138	352,616,061
01/30/09	587,693	-30,204,157	321,824,211
02/27/09	536,374	-35,378,530	285,909,307
03/31/09	476,516	24,417,944	309,850,735
04/30/09	516,418	29,102,754	338,437,071
05/29/09	564,062	17,964,723	355,837,732
06/30/09	593,063	69,686	355,314,355
07/31/09	592,191	26,343,631	381,065,795
08/31/09	635,110	12,788,634	393,219,319
09/30/09	655,366	14,047,125	406,611,078
10/30/09	677,685	-8,035,442	397,897,951
11/30/09	663,163	22,825,017	420,059,804
12/31/09	700,100	7,464,714	426,824,419
01/29/10	711,374	-15,781,518	410,331,526
02/26/10	683,886	11,700,067	421,347,708
03/31/10	702,246	24,773,715	445,419,177
04/30/10	742,365	6,574,087	451,250,899
05/28/10	752,085	-36,991,706	413,507,108
06/30/10	689,179	-22,280,746	390,537,184
07/30/10	650,895	26,860,301	416,746,589
08/31/10	694,578	-19,774,278	396,277,734
09/30/10	660,463	34,694,553	430,311,824
10/29/10	717,186	15,859,547	445,454,185
11/30/10	742,424	-1,020,216	443,691,546
12/31/10	739,486	28,973,090	471,925,150
01/31/11	786,542	10,687,024	481,825,631
02/28/11	803,043	15,397,501	496,420,089
03/31/11	827,367	-519,902	495,072,821
04/29/11	825,121	14,107,277	508,354,976
05/31/11	847,258	-6,863,264	500,644,454
06/30/11	834,407	-9,140,520	490,669,527
07/29/11	817,783	-10,536,852	479,314,892
08/31/11	798,858	-27,220,819	451,295,215
09/30/11	752,159	-32,385,853	418,157,203
10/31/11	696,929	45,045,164	462,505,439
11/30/11	770,842	-2,339,651	459,394,946
12/30/11	765,658	3,919,903	462,549,191
01/31/12	770,915	20,159,288	481,937,564
02/29/12	803,229	19,561,581	500,695,915
03/30/12	834,493	15,687,993	515,549,415
04/30/12	859,249	-3,865,330	510,824,835
05/31/12	851,375	-32,003,519	477,969,942
06/29/12	796,617	18,906,063	496,079,389
07/31/12	826,799	6,249,429	501,502,019

08/31/12	835,837	9,911,366	510,577,548
09/28/12	850,963	12,374,404	522,100,989
10/31/12	870,168	-10,332,067	510,898,753
11/30/12	851,498	1,454,377	511,501,632
12/31/12	852,503	3,615,452	514,264,582
01/31/13	857,108	25,933,367	539,340,842
02/28/13	898,901	5,965,435	544,407,375
03/28/13	907,346	19,592,023	563,092,053
04/30/13	938,487	10,183,950	572,337,516
05/31/13	953,896	11,883,320	583,266,940
06/28/13	972,112	-8,748,611	573,546,218
07/31/13	955,910	28,368,807	600,959,114
08/30/13	1,001,599	-18,808,826	581,148,689
09/30/13	968,581	17,288,868	597,468,977
10/31/13	995,782	26,644,583	623,117,778
11/29/13	1,038,530	17,478,118	639,557,366
12/31/13	1,065,929	15,069,784	653,561,221
01/31/14	1,089,269	-23,255,600	629,216,352
02/28/14	1,048,694	27,129,945	655,297,603
03/31/14	1,092,163	4,542,626	658,748,066
04/30/14	1,097,913	4,084,763	661,734,916
05/30/14	1,102,892	13,916,472	674,548,496
06/30/14	1,124,247	12,855,757	686,280,005
07/31/14	1,143,800	-10,349,009	674,787,197
08/29/14	1,124,645	25,409,329	699,071,881
09/30/14	1,165,120	-10,845,303	687,061,458
10/31/14	1,145,102	15,940,826	701,857,182
11/28/14	1,169,762	17,219,072	717,906,492
12/31/14	1,196,511	-3,006,960	713,703,022
01/30/15	1,189,505	-22,153,946	690,359,570
02/27/15	1,150,599	37,895,567	727,104,538
03/31/15	1,211,841	-12,648,751	713,243,945
04/30/15	1,188,740	6,077,382	718,132,588

Figure 3. 2% Spending, based on prior month's endowment balance

Spend 5%/12 of Prior Month's Balance			
Month End	Spend	Return	Balance
05/31/05			500,000,000
06/30/05	2,083,333	-71,339	497,845,328
07/29/05	2,074,356	17,906,602	513,677,575
08/31/05	2,140,323	-5,764,503	505,772,748
09/30/05	2,107,386	3,514,585	507,179,946
10/31/05	2,113,250	-8,997,748	496,068,948
11/30/05	2,066,954	17,454,742	511,456,737
12/30/05	2,131,070	-487,109	508,838,557
01/31/06	2,120,161	12,958,509	519,676,906
02/28/06	2,165,320	235,464	517,747,050
03/31/06	2,157,279	5,728,668	521,318,438
04/28/06	2,172,160	6,353,270	525,499,548
05/31/06	2,189,581	-16,246,818	507,063,149
06/30/06	2,112,763	43,916	504,994,302
07/31/06	2,104,143	2,568,307	505,458,465

08/31/06	2,106,077	10,753,256	514,105,645
09/29/06	2,142,107	12,629,660	524,593,198
10/31/06	2,185,805	16,528,897	538,936,291
11/30/06	2,245,568	8,874,453	545,565,176
12/29/06	2,273,188	6,882,715	550,174,703
01/31/07	2,292,395	7,734,953	555,617,261
02/28/07	2,315,072	-12,138,095	541,164,094
03/30/07	2,254,850	5,400,793	544,310,036
04/30/07	2,267,958	23,563,553	565,605,631
05/31/07	2,356,690	18,410,027	581,658,968
06/29/07	2,423,579	-10,363,016	568,872,373
07/31/07	2,370,302	-18,193,623	548,308,448
08/31/07	2,284,619	7,053,216	553,077,046
09/28/07	2,304,488	19,796,840	570,569,398
10/31/07	2,377,372	8,457,171	576,649,197
11/30/07	2,402,705	-25,397,605	548,848,887
12/31/07	2,286,870	-4,735,737	541,826,280
01/31/08	2,257,609	-33,139,978	506,428,692
02/28/08	2,110,120	-3,993,239	500,325,334
03/31/08	2,084,689	-16,454,605	481,786,040
04/30/08	2,007,442	22,907,329	502,685,927
05/30/08	2,094,525	5,365,747	505,957,149
06/30/08	2,108,155	-43,493,282	460,355,713
07/31/08	1,918,149	-4,538,820	453,898,744
08/29/08	1,891,245	5,533,254	457,540,754
09/30/08	1,906,420	-41,540,790	414,093,544
10/31/08	1,725,390	-70,157,606	342,210,548
11/28/08	1,425,877	-25,614,128	315,170,543
12/31/08	1,313,211	2,465,127	316,322,459
01/30/09	1,318,010	-27,095,343	287,909,106
02/27/09	1,199,621	-31,650,201	255,059,284
03/31/09	1,062,747	21,783,213	275,779,750
04/30/09	1,149,082	25,902,634	300,533,301
05/29/09	1,252,222	15,952,736	315,233,815
06/30/09	1,313,474	61,734	313,982,075
07/31/09	1,308,259	23,279,183	335,952,999
08/31/09	1,399,804	11,274,641	345,827,836
09/30/09	1,440,949	12,354,141	356,741,027
10/30/09	1,486,421	-7,049,911	348,204,695
11/30/09	1,450,853	19,974,413	366,728,255
12/31/09	1,528,034	6,516,980	371,717,201
01/29/10	1,548,822	-13,743,969	356,424,410
02/26/10	1,485,102	10,162,976	365,102,285
03/31/10	1,521,260	21,466,688	385,047,713
04/30/10	1,604,365	5,683,045	389,126,393
05/28/10	1,621,360	-31,898,993	355,606,041
06/30/10	1,481,692	-19,160,899	334,963,450
07/30/10	1,395,681	23,038,060	356,605,829
08/31/10	1,485,858	-16,920,649	338,199,323
09/30/10	1,409,164	29,609,724	366,399,883
10/29/10	1,526,666	13,504,013	378,377,229
11/30/10	1,576,572	-866,591	375,934,067
12/31/10	1,566,392	24,548,522	398,916,196
01/31/11	1,662,151	9,033,693	406,287,738
02/28/11	1,692,866	12,983,568	417,578,440

03/31/11	1,739,910	-437,331	415,401,199
04/29/11	1,730,838	11,837,006	425,507,367
05/31/11	1,772,947	-5,744,744	417,989,675
06/30/11	1,741,624	-7,631,450	408,616,602
07/29/11	1,702,569	-8,774,811	398,139,221
08/31/11	1,658,913	-22,610,763	373,869,544
09/30/11	1,557,790	-26,829,631	345,482,123
10/31/11	1,439,509	37,216,384	381,258,999
11/30/11	1,588,579	-1,928,654	377,741,766
12/30/11	1,573,924	3,223,177	379,391,018
01/31/12	1,580,796	16,535,005	394,345,227
02/29/12	1,643,105	16,006,256	408,708,378
03/30/12	1,702,952	12,805,805	419,811,231
04/30/12	1,749,213	-3,147,534	414,914,484
05/31/12	1,728,810	-25,994,671	387,191,002
06/29/12	1,613,296	15,315,310	400,893,016
07/31/12	1,670,388	5,050,306	404,272,934
08/31/12	1,684,471	7,989,792	410,578,256
09/28/12	1,710,743	9,950,812	418,818,325
10/31/12	1,745,076	-8,288,165	408,785,084
11/30/12	1,703,271	1,163,690	408,245,502
12/31/12	1,701,023	2,885,606	409,430,085
01/31/13	1,705,959	20,646,766	428,370,893
02/28/13	1,784,879	4,738,040	431,324,054
03/28/13	1,797,184	15,522,404	445,049,274
04/30/13	1,854,372	8,049,056	451,243,959
05/31/13	1,880,183	9,369,081	458,732,856
06/28/13	1,911,387	-6,880,683	449,940,786
07/31/13	1,874,753	22,255,021	470,321,054
08/30/13	1,959,671	-14,720,115	453,641,268
09/30/13	1,890,172	13,495,590	465,246,686
10/31/13	1,938,528	20,748,029	484,056,187
11/29/13	2,016,901	13,577,515	495,616,801
12/31/13	2,065,070	11,678,136	505,229,868
01/31/14	2,105,124	-17,977,541	485,147,202
02/28/14	2,021,447	20,918,110	504,043,865
03/31/14	2,100,183	3,494,111	505,437,794
04/30/14	2,105,991	3,134,117	506,465,920
05/30/14	2,110,275	10,651,121	515,006,767
06/30/14	2,145,862	9,815,160	522,676,065
07/31/14	2,177,817	-7,881,884	512,616,365
08/29/14	2,135,902	19,302,734	529,783,198
09/30/14	2,207,430	-8,218,982	519,356,786
10/31/14	2,163,987	12,049,834	529,242,633
11/28/14	2,205,178	12,984,219	540,021,674
12/31/14	2,250,090	-2,261,887	535,509,696
01/30/15	2,231,290	-16,622,675	516,655,731
02/27/15	2,152,732	28,360,528	542,863,527
03/31/15	2,261,931	-9,443,684	531,157,911
04/30/15	2,213,158	4,525,871	533,470,624

Figure 4. 5% Spending, based on prior month's endowment balance

Spend 8%/12 of Prior Month's Balance

Month End	Spend	Return	Balance
05/31/05			500,000,000
06/30/05	3,333,333	-71,339	496,595,328
07/29/05	3,310,636	17,861,642	511,146,334
08/31/05	3,407,642	-5,736,097	502,002,595
09/30/05	3,346,684	3,488,386	502,144,297
10/31/05	3,347,629	-8,908,412	489,888,256
11/30/05	3,265,922	17,237,267	503,859,602
12/30/05	3,359,064	-479,874	500,020,664
01/31/06	3,333,471	12,733,946	509,421,138
02/28/06	3,396,141	230,817	506,255,814
03/31/06	3,375,039	5,601,522	508,482,297
04/28/06	3,389,882	6,196,837	511,289,253
05/31/06	3,408,595	-15,807,479	492,073,178
06/30/06	3,280,488	42,617	488,835,308
07/31/06	3,258,902	2,486,125	488,062,531
08/31/06	3,253,750	10,383,170	495,191,951
09/29/06	3,301,280	12,165,021	504,055,693
10/31/06	3,360,371	15,881,801	516,577,123
11/30/06	3,443,847	8,506,274	521,639,549
12/29/06	3,477,597	6,580,875	524,742,827
01/31/07	3,498,286	7,377,404	528,621,945
02/28/07	3,524,146	-11,548,352	513,549,447
03/30/07	3,423,663	5,125,200	515,250,985
04/30/07	3,435,007	22,305,567	534,121,545
05/31/07	3,560,810	17,385,244	547,945,979
06/29/07	3,652,973	-9,762,375	534,530,631
07/31/07	3,563,538	-17,095,309	513,871,784
08/31/07	3,425,812	6,610,237	517,056,209
09/28/07	3,447,041	18,507,511	532,116,679
10/31/07	3,547,445	7,887,212	536,456,446
11/30/07	3,576,376	-23,627,379	509,252,691
12/31/07	3,395,018	-4,394,081	501,463,592
01/31/08	3,343,091	-30,671,256	467,449,245
02/28/08	3,116,328	-3,685,882	460,647,035
03/31/08	3,070,980	-15,149,672	442,426,382
04/30/08	2,949,509	21,035,908	460,512,781
05/30/08	3,070,085	4,915,584	462,358,279
06/30/08	3,082,389	-39,745,419	419,530,472
07/31/08	2,796,870	-4,136,308	412,597,294
08/29/08	2,750,649	5,029,769	414,876,414
09/30/08	2,765,843	-37,667,233	374,443,339
10/31/08	2,496,289	-63,439,888	308,507,161
11/28/08	2,056,714	-23,091,462	283,358,985
12/31/08	1,889,060	2,216,311	283,686,236
01/30/09	1,891,242	-24,299,811	257,495,183
02/27/09	1,716,635	-28,306,761	227,471,788
03/31/09	1,516,479	19,427,116	245,382,425
04/30/09	1,635,883	23,047,563	266,794,105
05/29/09	1,778,627	14,161,812	279,177,290
06/30/09	1,861,182	54,673	277,370,781
07/31/09	1,849,139	20,564,757	296,086,399
08/31/09	1,973,909	9,936,711	304,049,201

09/30/09	2,026,995	10,861,666	312,883,872
10/30/09	2,085,892	-6,183,207	304,614,773
11/30/09	2,030,765	17,473,921	320,057,929
12/31/09	2,133,720	5,687,621	323,611,830
01/29/10	2,157,412	-11,965,309	309,489,109
02/26/10	2,063,261	8,824,678	316,250,526
03/31/10	2,108,337	18,594,382	332,736,571
04/30/10	2,218,244	4,910,968	335,429,295
05/28/10	2,236,195	-27,497,124	305,695,976
06/30/10	2,037,973	-16,471,626	287,186,377
07/30/10	1,914,576	19,752,057	305,023,858
08/31/10	2,033,492	-14,473,127	288,517,238
09/30/10	1,923,448	25,260,003	311,853,793
10/29/10	2,079,025	11,493,665	321,268,433
11/30/10	2,141,790	-735,796	318,390,848
12/31/10	2,122,606	20,790,945	337,059,187
01/31/11	2,247,061	7,632,904	342,445,030
02/28/11	2,282,967	10,943,373	351,105,436
03/31/11	2,340,703	-367,713	348,397,020
04/29/11	2,322,647	9,927,698	356,002,071
05/31/11	2,373,347	-4,806,358	348,822,365
06/30/11	2,325,482	-6,368,627	340,128,256
07/29/11	2,267,522	-7,304,063	330,556,671
08/31/11	2,203,711	-18,772,676	309,580,284
09/30/11	2,063,869	-22,216,104	285,300,311
10/31/11	1,902,002	30,733,416	314,131,725
11/30/11	2,094,212	-1,589,081	310,448,433
12/30/11	2,069,656	2,648,979	311,027,756
01/31/12	2,073,518	13,555,527	322,509,765
02/29/12	2,150,065	13,090,494	333,450,194
03/30/12	2,223,001	10,447,787	341,674,980
04/30/12	2,277,833	-2,561,707	336,835,439
05/31/12	2,245,570	-21,102,966	313,486,903
06/29/12	2,089,913	12,399,950	323,796,940
07/31/12	2,158,646	4,079,077	325,717,371
08/31/12	2,171,449	6,437,270	329,983,192
09/28/12	2,199,888	7,997,502	335,780,807
10/31/12	2,238,539	-6,644,902	326,897,366
11/30/12	2,179,316	930,580	325,648,630
12/31/12	2,170,991	2,301,786	325,779,425
01/31/13	2,171,863	16,428,426	340,035,987
02/28/13	2,266,907	3,761,003	341,530,084
03/28/13	2,276,867	12,290,916	351,544,133
04/30/13	2,343,628	6,357,944	355,558,450
05/31/13	2,370,390	7,382,383	360,570,443
06/28/13	2,403,803	-5,408,313	352,758,327
07/31/13	2,351,722	17,448,172	367,854,776
08/30/13	2,452,365	-11,513,124	353,889,287
09/30/13	2,359,262	10,528,020	362,058,046
10/31/13	2,413,720	16,146,254	375,790,579
11/29/13	2,505,271	10,540,723	383,826,032
12/31/13	2,558,840	9,044,029	390,311,220
01/31/14	2,602,075	-13,888,403	373,820,742
02/28/14	2,492,138	16,118,043	387,446,647
03/31/14	2,582,978	2,685,841	387,549,510

04/30/14	2,583,663	2,403,116	387,368,963
05/30/14	2,582,460	8,146,479	392,932,982
06/30/14	2,619,553	7,488,640	397,802,068
07/31/14	2,652,014	-5,998,801	389,151,254
08/29/14	2,594,342	14,653,616	401,210,528
09/30/14	2,674,737	-6,224,324	392,311,467
10/31/14	2,615,410	9,102,197	398,798,255
11/28/14	2,658,655	9,783,951	405,923,551
12/31/14	2,706,157	-1,700,216	401,517,178
01/30/15	2,676,781	-12,463,433	386,376,963
02/27/15	2,575,846	21,209,200	405,010,317
03/31/15	2,700,069	-7,045,582	395,264,666
04/30/15	2,635,098	3,367,956	395,997,524

Figure 5. 8% Spending, based on prior month's endowment balance

Spend 2% of Prior Year End Balance

Month End	Spend	Return	Balance
05/31/05			
06/30/05			
07/29/05			
08/31/05			
09/30/05			
10/31/05			
11/30/05			
12/30/05			
01/31/06			
02/28/06			
03/31/06			
04/28/06			
05/31/06			
06/30/06			533,025,598
07/31/06	888,376	2,710,869	534,848,091
08/31/06	888,376	11,378,499	545,338,213
09/29/06	888,376	13,396,928	557,846,766
10/31/06	888,376	17,576,652	574,535,041
11/30/06	888,376	9,460,644	583,107,310
12/29/06	888,376	7,356,337	589,575,271
01/31/07	888,376	8,288,889	596,975,783
02/28/07	888,376	-13,041,620	583,045,788
03/30/07	888,376	5,818,771	587,976,182
04/30/07	888,376	25,453,891	612,541,697
05/31/07	888,376	19,937,760	631,591,081
06/29/07	888,376	-11,252,622	619,450,082
07/31/07	1,032,417	-19,811,195	598,606,470
08/31/07	1,032,417	7,700,230	605,274,283
09/28/07	1,032,417	21,665,188	625,907,055
10/31/07	1,032,417	9,277,404	634,152,042
11/30/07	1,032,417	-27,930,227	605,189,398
12/31/07	1,032,417	-5,221,870	598,935,111
01/31/08	1,032,417	-36,632,953	561,269,742
02/28/08	1,032,417	-4,425,666	555,811,659
03/31/08	1,032,417	-18,279,428	536,499,814

04/30/08	1,032,417	25,508,788	560,976,185
05/30/08	1,032,417	5,987,946	565,931,714
06/30/08	1,032,417	-48,648,838	516,250,459
07/31/08	860,417	-5,089,907	510,300,135
08/29/08	860,417	6,220,815	515,660,533
09/30/08	860,417	-46,817,569	467,982,546
10/31/08	860,417	-79,287,725	387,834,404
11/28/08	860,417	-29,029,030	357,944,956
12/31/08	860,417	2,799,690	359,884,229
01/30/09	860,417	-30,826,729	328,197,083
02/27/09	860,417	-36,079,107	291,257,558
03/31/09	860,417	24,874,709	315,271,849
04/30/09	860,417	29,611,932	344,023,364
05/29/09	860,417	18,261,251	361,424,197
06/30/09	860,417	70,780	360,634,560
07/31/09	601,058	26,738,080	386,771,582
08/31/09	601,058	12,980,121	399,150,646
09/30/09	601,058	14,259,012	412,808,600
10/30/09	601,058	-8,157,918	404,049,624
11/30/09	601,058	23,177,901	426,626,468
12/31/09	601,058	7,581,407	433,606,818
01/29/10	601,058	-16,032,292	416,973,468
02/26/10	601,058	11,889,454	428,261,864
03/31/10	601,058	25,180,242	452,841,049
04/30/10	601,058	6,683,629	458,923,620
05/28/10	601,058	-37,620,684	420,701,878
06/30/10	601,058	-22,668,417	397,432,404
07/30/10	662,387	27,334,539	424,104,556
08/31/10	662,387	-20,123,407	403,318,761
09/30/10	662,387	35,311,003	437,967,377
10/29/10	662,387	16,141,700	453,446,689
11/30/10	662,387	-1,038,521	451,745,781
12/31/10	662,387	29,499,032	480,582,426
01/31/11	662,387	10,883,073	490,803,111
02/28/11	662,387	15,684,390	505,825,114
03/31/11	662,387	-529,752	504,632,975
04/29/11	662,387	14,379,697	518,350,284
05/31/11	662,387	-6,998,210	510,689,687
06/30/11	662,387	-9,323,921	500,703,379
07/29/11	834,506	-10,752,323	489,116,550
08/31/11	834,506	-27,777,466	460,504,579
09/30/11	834,506	-33,046,736	426,623,338
10/31/11	834,506	45,957,162	471,745,994
11/30/11	834,506	-2,386,396	468,525,093
12/30/11	834,506	3,997,808	471,688,395
01/31/12	834,506	20,557,603	491,411,492
02/29/12	834,506	19,946,122	510,523,109
03/30/12	834,506	15,995,902	525,684,506
04/30/12	834,506	-3,941,318	520,908,682
05/31/12	834,506	-32,635,279	487,438,898
06/29/12	834,506	19,280,607	505,884,999
07/31/12	843,142	6,372,957	511,414,814
08/31/12	843,142	10,107,276	520,678,948
09/28/12	843,142	12,619,222	532,455,029
10/31/12	843,142	-10,536,967	521,074,920

11/30/12	843,142	1,483,345	521,715,123
12/31/12	843,142	3,687,644	524,559,626
01/31/13	843,142	26,452,526	550,169,011
02/28/13	843,142	6,085,201	555,411,070
03/28/13	843,142	19,988,022	574,555,951
04/30/13	843,142	10,391,283	584,104,092
05/31/13	843,142	12,127,627	595,388,578
06/28/13	843,142	-8,930,427	585,615,009
07/31/13	976,025	28,965,755	613,604,738
08/30/13	976,025	-19,204,609	593,424,104
09/30/13	976,025	17,654,055	610,102,135
10/31/13	976,025	27,207,968	636,334,078
11/29/13	976,025	17,848,828	653,206,881
12/31/13	976,025	15,391,405	667,622,261
01/31/14	976,025	-23,755,933	642,890,303
02/28/14	976,025	27,719,525	669,633,803
03/31/14	976,025	4,642,007	673,299,785
04/30/14	976,025	4,174,995	676,498,755
05/30/14	976,025	14,226,960	689,749,690
06/30/14	976,025	13,145,466	701,919,130
07/31/14	1,169,865	-10,584,844	690,164,421
08/29/14	1,169,865	25,988,363	714,982,919
09/30/14	1,169,865	-11,092,144	702,720,909
10/31/14	1,169,865	16,304,148	717,855,193
11/28/14	1,169,865	17,611,561	734,296,888
12/31/14	1,169,865	-3,075,611	730,051,412
01/30/15	1,169,865	-22,661,414	706,220,132
02/27/15	1,169,865	38,766,193	743,816,460
03/31/15	1,169,865	-12,939,473	729,707,122
04/30/15	1,169,865	6,217,661	734,754,918

Figure 6. 2% Spending, based on prior year's endowment balance

Spend 5% of Prior Year End Balance			
Month End	Spend	Return	Balance
05/31/05			
06/30/05			
07/29/05			
08/31/05			
09/30/05			
10/31/05			
11/30/05			
12/30/05			
01/31/06			
02/28/06			
03/31/06			
04/28/06			
05/31/06			
06/30/06			533,025,598
07/31/06	2,220,940	2,710,869	533,515,527
08/31/06	2,220,940	11,350,149	542,644,736
09/29/06	2,220,940	13,330,760	553,754,556
10/31/06	2,220,940	17,447,714	568,981,330

11/30/06	2,220,940	9,369,193	576,129,583
12/29/06	2,220,940	7,268,308	581,176,951
01/31/07	2,220,940	8,170,816	587,126,827
02/28/07	2,220,940	-12,826,458	572,079,429
03/30/07	2,220,940	5,709,327	575,567,816
04/30/07	2,220,940	24,916,724	598,263,600
05/31/07	2,220,940	19,473,019	615,515,679
06/29/07	2,220,940	-10,966,218	602,328,521
07/31/07	2,509,702	-19,263,615	580,555,204
08/31/07	2,509,702	7,468,025	585,513,527
09/28/07	2,509,702	20,957,872	603,961,697
10/31/07	2,509,702	8,952,123	610,404,117
11/30/07	2,509,702	-26,884,287	581,010,128
12/31/07	2,509,702	-5,013,239	573,487,186
01/31/08	2,509,702	-35,076,469	535,901,015
02/28/08	2,509,702	-4,225,631	529,165,681
03/31/08	2,509,702	-17,403,100	509,252,879
04/30/08	2,509,702	24,213,286	530,956,463
05/30/08	2,509,702	5,667,511	534,114,271
06/30/08	2,509,702	-45,913,735	485,690,834
07/31/08	2,023,712	-4,788,608	478,878,514
08/29/08	2,023,712	5,837,770	482,692,573
09/30/08	2,023,712	-43,824,360	436,844,501
10/31/08	2,023,712	-74,012,176	360,808,613
11/28/08	2,023,712	-27,006,175	331,778,725
12/31/08	2,023,712	2,595,029	332,350,043
01/30/09	2,023,712	-28,468,223	301,858,108
02/27/09	2,023,712	-33,183,631	266,650,764
03/31/09	2,023,712	22,773,177	287,400,230
04/30/09	2,023,712	26,994,088	312,370,606
05/29/09	2,023,712	16,581,077	326,927,972
06/30/09	2,023,712	64,024	324,968,284
07/31/09	1,354,035	24,093,720	347,707,969
08/31/09	1,354,035	11,669,140	358,023,074
09/30/09	1,354,035	12,789,796	369,458,836
10/30/09	1,354,035	-7,301,240	360,803,561
11/30/09	1,354,035	20,697,134	380,146,661
12/31/09	1,354,035	6,755,433	385,548,059
01/29/10	1,354,035	-14,255,355	369,938,670
02/26/10	1,354,035	10,548,318	379,132,953
03/31/10	1,354,035	22,291,640	400,070,559
04/30/10	1,354,035	5,904,772	404,621,297
05/28/10	1,354,035	-33,169,202	370,098,061
06/30/10	1,354,035	-19,941,763	348,802,263
07/30/10	1,453,343	23,989,864	371,338,784
08/31/10	1,453,343	-17,619,715	352,265,726
09/30/10	1,453,343	30,841,253	381,653,637
10/29/10	1,453,343	14,066,204	394,266,498
11/30/10	1,453,343	-902,982	391,910,173
12/31/10	1,453,343	25,591,763	416,048,593
01/31/11	1,453,343	9,421,666	424,016,916
02/28/11	1,453,343	13,550,132	436,113,705
03/31/11	1,453,343	-456,743	434,203,620
04/29/11	1,453,343	12,372,787	445,123,064
05/31/11	1,453,343	-6,009,574	437,660,147

06/30/11	1,453,343	-7,990,584	428,216,221
07/29/11	1,784,234	-9,195,702	417,236,284
08/31/11	1,784,234	-23,695,307	391,756,743
09/30/11	1,784,234	-28,113,253	361,859,257
10/31/11	1,784,234	38,980,579	399,055,601
11/30/11	1,784,234	-2,018,681	395,252,686
12/30/11	1,784,234	3,372,593	396,841,045
01/31/12	1,784,234	17,295,529	412,352,340
02/29/12	1,784,234	16,737,155	427,305,260
03/30/12	1,784,234	13,388,489	438,909,515
04/30/12	1,784,234	-3,290,723	433,834,558
05/31/12	1,784,234	-27,180,026	404,870,298
06/29/12	1,784,234	16,014,613	419,100,676
07/31/12	1,746,253	5,279,679	422,634,102
08/31/12	1,746,253	8,352,671	429,240,520
09/28/12	1,746,253	10,403,112	437,897,379
10/31/12	1,746,253	-8,665,728	427,485,399
11/30/12	1,746,253	1,216,924	426,956,070
12/31/12	1,746,253	3,017,858	428,227,675
01/31/13	1,746,253	21,594,692	448,076,115
02/28/13	1,746,253	4,955,992	451,285,854
03/28/13	1,746,253	16,240,785	465,780,386
04/30/13	1,746,253	8,423,994	472,458,127
05/31/13	1,746,253	9,809,546	480,521,420
06/28/13	1,746,253	-7,207,497	471,567,670
07/31/13	1,964,865	23,324,733	492,927,537
08/30/13	1,964,865	-15,427,653	475,535,019
09/30/13	1,964,865	14,146,917	487,717,071
10/31/13	1,964,865	21,750,113	507,502,319
11/29/13	1,964,865	14,235,166	519,772,620
12/31/13	1,964,865	12,247,316	530,055,071
01/31/14	1,964,865	-18,860,894	509,229,311
02/28/14	1,964,865	21,956,459	529,220,906
03/31/14	1,964,865	3,668,643	530,924,683
04/30/14	1,964,865	3,292,156	532,251,974
05/30/14	1,964,865	11,193,409	541,480,518
06/30/14	1,964,865	10,319,705	549,835,358
07/31/14	2,290,981	-8,291,442	539,252,935
08/29/14	2,290,981	20,305,743	557,267,697
09/30/14	2,290,981	-8,645,373	546,331,344
10/31/14	2,290,981	12,675,683	556,716,046
11/28/14	2,290,981	13,658,240	568,083,305
12/31/14	2,290,981	-2,379,424	563,412,901
01/30/15	2,290,981	-17,488,814	543,633,107
02/27/15	2,290,981	29,841,383	571,183,509
03/31/15	2,290,981	-9,936,340	558,956,188
04/30/15	2,290,981	4,762,733	561,427,941

Figure 7. 5% Spending, based on prior year's endowment balance

Spend 8% of Prior Year End Balance			
Month End	Spend	Return	Balance
05/31/05			

Date			
06/30/05			
07/29/05			
08/31/05			
09/30/05			
10/31/05			
11/30/05			
12/30/05			
01/31/06			
02/28/06			
03/31/06			
04/28/06			
05/31/06			
06/30/06			533,025,598
07/31/06	3,553,504	2,710,869	532,182,963
08/31/06	3,553,504	11,321,800	539,951,259
09/29/06	3,553,504	13,264,591	549,662,346
10/31/06	3,553,504	17,318,777	563,427,619
11/30/06	3,553,504	9,277,743	569,151,857
12/29/06	3,553,504	7,180,278	572,778,632
01/31/07	3,553,504	8,052,743	577,277,871
02/28/07	3,553,504	-12,611,296	561,113,071
03/30/07	3,553,504	5,599,883	563,159,450
04/30/07	3,553,504	24,379,557	583,985,503
05/31/07	3,553,504	19,008,278	599,440,277
06/29/07	3,553,504	-10,679,814	585,206,959
07/31/07	3,901,380	-18,716,035	562,589,545
08/31/07	3,901,380	7,236,923	565,925,088
09/28/07	3,901,380	20,256,723	582,280,431
10/31/07	3,901,380	8,630,756	587,009,807
11/30/07	3,901,380	-25,853,922	557,254,506
12/31/07	3,901,380	-4,808,264	548,544,862
01/31/08	3,901,380	-33,550,910	511,092,572
02/28/08	3,901,380	-4,030,014	503,161,178
03/31/08	3,901,380	-16,547,869	482,711,929
04/30/08	3,901,380	22,951,352	501,761,901
05/30/08	3,901,380	5,355,883	503,216,405
06/30/08	3,901,380	-43,257,681	456,057,345
07/31/08	3,040,382	-4,496,440	448,520,522
08/29/08	3,040,382	5,467,691	450,947,830
09/30/08	3,040,382	-40,942,209	406,965,239
10/31/08	3,040,382	-68,949,896	334,974,961
11/28/08	3,040,382	-25,072,552	306,862,027
12/31/08	3,040,382	2,400,141	306,221,786
01/30/09	3,040,382	-26,230,146	276,951,258
02/27/09	3,040,382	-30,445,591	243,465,284
03/31/09	3,040,382	20,793,033	261,217,935
04/30/09	3,040,382	24,534,914	282,712,467
05/29/09	3,040,382	15,006,781	294,678,866
06/30/09	3,040,382	57,709	291,696,192
07/31/09	1,944,641	21,626,868	311,378,419
08/31/09	1,944,641	10,449,914	319,883,691
09/30/09	1,944,641	11,427,328	329,366,378
10/30/09	1,944,641	-6,508,934	320,912,803
11/30/09	1,944,641	18,408,841	337,377,002
12/31/09	1,944,641	5,995,391	341,427,752

Date			
01/29/10	1,944,641	-12,624,039	326,859,072
02/26/10	1,944,641	9,319,959	334,234,390
03/31/10	1,944,641	19,651,768	351,941,516
04/30/10	1,944,641	5,194,420	355,191,295
05/28/10	1,944,641	-29,117,132	324,129,522
06/30/10	1,944,641	-17,464,869	304,720,012
07/30/10	2,031,467	20,957,982	323,646,527
08/31/10	2,031,467	-15,356,757	306,258,303
09/30/10	2,031,467	26,813,253	331,040,089
10/29/10	2,031,467	12,200,794	341,209,416
11/30/10	2,031,467	-781,466	338,396,483
12/31/10	2,031,467	22,097,315	358,462,331
01/31/11	2,031,467	8,117,591	364,548,455
02/28/11	2,031,467	11,649,723	374,166,711
03/31/11	2,031,467	-391,866	371,743,379
04/29/11	2,031,467	10,592,961	380,304,873
05/31/11	2,031,467	-5,134,469	373,138,938
06/30/11	2,031,467	-6,812,587	364,294,884
07/29/11	2,428,633	-7,823,027	354,043,224
08/31/11	2,428,633	-20,106,503	331,508,088
09/30/11	2,428,633	-23,789,688	305,289,768
10/31/11	2,428,633	32,886,741	335,747,876
11/30/11	2,428,633	-1,698,429	331,620,814
12/30/11	2,428,633	2,829,638	332,021,820
01/31/12	2,428,633	14,470,512	344,063,699
02/29/12	2,428,633	13,965,356	355,600,423
03/30/12	2,428,633	11,141,806	364,313,597
04/30/12	2,428,633	-2,731,440	359,153,524
05/31/12	2,428,633	-22,501,209	334,223,682
06/29/12	2,428,633	13,220,191	345,015,241
07/31/12	2,300,102	4,346,377	347,061,517
08/31/12	2,300,102	6,859,102	351,620,518
09/28/12	2,300,102	8,521,907	357,842,323
10/31/12	2,300,102	-7,081,486	348,460,735
11/30/12	2,300,102	991,964	347,152,597
12/31/12	2,300,102	2,453,782	347,306,278
01/31/13	2,300,102	17,513,983	362,520,160
02/28/13	2,300,102	4,009,692	364,229,750
03/28/13	2,300,102	13,107,827	375,037,475
04/30/13	2,300,102	6,782,839	379,520,213
05/31/13	2,300,102	7,879,896	385,100,007
06/28/13	2,300,102	-5,776,240	377,023,665
07/31/13	2,513,491	18,648,386	393,158,561
08/30/13	2,513,491	-12,305,082	378,339,988
09/30/13	2,513,491	11,255,416	387,081,913
10/31/13	2,513,491	17,262,212	401,830,634
11/29/13	2,513,491	11,271,133	410,588,275
12/31/13	2,513,491	9,674,623	417,749,407
01/31/14	2,513,491	-14,864,733	400,371,183
02/28/14	2,513,491	17,262,819	415,120,511
03/31/14	2,513,491	2,877,681	415,484,700
04/30/14	2,513,491	2,576,336	415,547,546
05/30/14	2,513,491	8,739,082	421,773,137
06/30/14	2,513,491	8,038,285	427,297,930
07/31/14	2,848,653	-6,443,594	418,005,683

08/29/14	2,848,653	15,740,138	430,897,169
09/30/14	2,848,653	-6,684,878	421,363,638
10/31/14	2,848,653	9,776,250	428,291,235
11/28/14	2,848,653	10,507,519	435,950,101
12/31/14	2,848,653	-1,825,982	431,275,466
01/30/15	2,848,653	-13,387,156	415,039,657
02/27/15	2,848,653	22,782,567	434,973,571
03/31/15	2,848,653	-7,566,825	424,558,094
04/30/15	2,848,653	3,617,559	425,327,000

Figure 8. 8% Spending, based on prior year's endowment balance

When the actual S&P 500 monthly returns are applied to the initial $500,000,000 endowment balance, using each of the spending rates applied to the prior month and prior year balances, respectively, the ending endowment balances are as follows:

	Applied to	
Spending Rate	Prior Month	Prior Year
0%	$875,161,561	$875,161,561
2%	$718,132,588	$734,754,918
5%	$533,470,624	$561,427,941
8%	$395,997,524	$425,327,000

We then attempted to develop a single function to estimate the ending endowment balance, making use of specific metrics generally considered crucial to the determination of a portfolio balance:

(1) the mean return

(2) the standard deviation of the returns (as a measure of volatility)

(3) the spending rate

We realized that it might be of some value to incorporate the spending pattern into the formula as well (that is, whether spending was based on the prior month's or rather the prior year's balance), but in this preliminary research wished to see how well the function performed without specifying the spending pattern.

A number of studies -- for example, Jacquier, Kane, and Marcios (2003), Blume (1974), and Indro and Lee (1997) have discussed the issue of whether one should use the arithmetic mean or instead the geometric mean (which takes compounding into account) when examining expected long-run rates of return and/or forecasting the terminal value of a portfolio. Blitz and van Vliet (2007) conducted a study using twenty years' worth of data to demonstrate the effect of volatility on a portfolio balance, and Marx and Hargis (2014) discussed the effect of volatility as it affects investors taking withdrawals from a portfolio during retirement. Many researchers (among them Finke, Pfau, and Blanchett (2013), Vernon (2014), and Guyton (2004)) have concentrated specifically on the effect of spending from a portfolio, usually in the context of systematic withdrawals for retirement income.

There was some trial and error involved in the development of the estimating function ultimately used, especially as it related to the utilization of a constant, attached as a weight to the average return. The formula ultimately used to make our predictions of the final balances is as follows:

$$EB = (BB * (1 + (SD / C * MR) * .01) - SR) \wedge (N/12)$$

where

EB = ending portfolio balance

BB = beginning portfolio balance

SD = standard deviation of the monthly returns

MR = arithmetic mean of the monthly returns

SR = spending rate

N = number of periods

C = constant

Note that the formula above indeed does incorporate the mean return, the standard deviation of the returns, and the spending rate. In fact, it makes use of a modified version of the coefficient of variation (standard deviation/mean), used to measure relative volatility. Here a weight or adjustment factor was applied to the mean. The weight used (C =.4125) was the one that provided the "best fit" (based on trial and error) in terms of how close the predicted final portfolio balances using the formula were to what the actual portfolio balances would have been. Note that "best fit" is in quotations because we did not determine the optimal constant value based on a specific statistical test for goodness of fit.

The calculated ending balances using our function compared to the actual ending balances when spending is based on the prior month's portfolio balance are as follows:

Spending Rate	Actual	Calculated
0%	$875,161,561	$877,361,623
2%	$718,132,588	$726,127,708
5%	$533,470,624	$542,935,738
8%	$395,997,524	$402,411,100

The calculated ending balances using the function compared to the actual ending balances when spending is based on the prior year's portfolio balance are as follows:

Spending Rate	Actual	Calculated
0%	$875,161,561	$877,361,623
2%	$734,754,918	$726,127,708
5%	$561,427,941	$542,935,738
8%	$425,327,000	$402,411,100

3. Analysis

The impetus for this research was the suggestion that an institution that bases its spending on a prior month's or prior year's balance will be harmed when markets are volatile, because it may be spending based on a higher previous portfolio balance at a time when a market down cycle is further eroding the corpus. That would make it more difficult for the portfolio to recover to prior levels (the math of investing is basic to all investors – if the portfolio drops 50%, then it needs to increase by 100% to get back to where it started. If the portfolio increases by 50%, then it only needs to drop by 33% get back to the starting point). An examination of the formula used in this study (which seems to do a decent job of estimating final portfolio balances) would lead one to a different conclusion regarding volatility, as evidenced by the predicted ending balances being rather good estimates of the actual, without consideration of the smoothing of spending.

Relationships clearly change over time. Therefore, the adjustment factor (applied to the average return) providing the best fit for a different time period would be expected to vary from the one for the ten year period under consideration here. However, if the general form of the relationship is shown to be robust over time, investors may well be able to be less concerned about volatility (as measured by the standard deviation) than they currently are.

Given that the formula does not include a variable that takes into account the spending pattern, but rather only one for the spending rate, the formula predicts the same ending balance for both patterns used (spending based on the prior month's balance and spending based on the prior year's balance). Therefore, if the formula is indeed a decent predictor in both cases, then perhaps the spending pattern makes little difference in the ending portfolio balance. However, clearly more research would be required using broader differences in the spending pattern to see the extent to which that result holds.

4. Additional Test

In order to test whether the formula is indeed robust, we took the data set and arbitrarily truncated the data. The original data set had 119 observations. This was truncated to 98 observations for spending based on the prior year's balance and 88 observations for spending based on the prior month's balance. Then the formula described earlier in this paper was applied. The arithmetic mean return and standard deviation of the returns were 0.5111% and 4.6314%,

respectively, for spending based on the prior year's balance and 0.4947% and 4.8075% for spending based on the prior month's balance.

The calculated ending balances employing our function (still using the original adjustment factor of .4125) compared to the actual ending balances when spending is based on the prior month's portfolio balance are as follows:

Spending Rate	Actual	Calculated
0%	$783,076,676	$754,211,200
2%	$667,009,974	$655,680,083
5%	$535,526,649	$528,758,210
8%	$429,725,728	$423,641,059

The calculated ending balances using the function compared to the actual ending balances when spending is based on the prior year's portfolio balance are as follows:

Spending Rate	Actual	Calculated
0%	$824,032,894	$788,627,972
2%	$702,447,450	$674,752,489
5%	$549,609,947	$530,963,277
8%	$426,825,583	$414,798,943

It should be noted that a modest change of the adjustment factor from .4125 to .38 for the truncated data results in the ending balance being somewhere between 98% and 101% of the predicted balance for all eight scenarios.

5. Conclusions

Fiduciaries who are entrusted with the management of an endowment have an obligation, moral and legal, to manage that endowment to the goals specified in the Investment Policy Statement. Often, the first goal is "perpetuity." Further goals may include absolute return targets (say 6%), and/or relative return targets (S&P 500 plus 5% or top quartile among endowments of $1 billion and over). A critical estimate is the amount of the organization's spending budget. Frequently a rolling twelve quarters is used, with a set percentage taken for spending. This percentage can be regulatory (5% in the case of private foundations), contractual (built into the pension model) or organizationally specific (a University's spending policy). Using a set percentage based on the past might lead to spending that cannot be sustained, jeopardizing perpetuity. In a sustained market drop, spending will be based on the previous three years' average portfolio balance (where the rolling twelve quarters is used), resulting in more spending than the current portfolio balance can fund.

This paper develops a robust function that predicts the ending portfolio balance given a set of variable inputs. A fiduciary can adjust the variables to accommodate future scenarios, allowing them to see the resultant effect on the ending balance of the portfolio. This would allow for adjustments to controllable variables, such as spending, in advance of the permanent decline of the corpus.

As a method of obtaining a rough estimate of a portfolio's ending balance, the function developed in this study seems to work well. When applied to sixteen scenarios, the actual ending balances as a proportion of the predicted ending balances were (from lowest to highest):

0.98

0.98

0.99

1.00

1.00

1.01

1.01

1.01

1.02

1.03

1.03

1.04

1.04

1.04

1.04

1.06

The results above were obtained from our function by applying the same adjustment factor to the average return for all sixteen scenarios (which made use of returns for as few as 88 months to as many as 119 months). Therefore, we are in a sense providing a conservative estimate of the function's efficiency, given that one might expect to alter the adjustment factor (to obtain the best fit) when changing the time period under consideration. We see also that the function works well in each spending scenario, regardless as to whether the spending is calculated based on the prior month's or prior year's balance.

Any person or organization that has an investment fund, portfolio, or endowment where they have access to historical return mean and standard deviation, and have used a standard rate of spending can apply the function to see how well it estimates the ending balance. If it turns out to be a good predictor, the function can be used to estimate future balances based on the arithmetic mean return and the standard deviation of the returns. This could be useful, for instance, to predict how the portfolio will withstand an increase in the spending rate. Furthermore, as suggested earlier, confidence in the function presented here could lead to a decrease in one's trepidation about volatility of returns.

Future research should incorporate the spending pattern as part of the formula. This would perhaps yield forecasted balances that were even closer to the observed actual balances. It might also be of value to develop a series of return mean-standard deviation pairings to understand how a change in one might result in the change in the other. A casual look at our formula might lead one to conclude that the ending portfolio balance can be increased merely by increasing the standard deviation. This would be an ill-advised implication, given that the required condition "holding all else constant" is not possible – return mean and the standard deviation of returns are linked.

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