The Size Effect and the Value Effect in the American Stock Market

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Abstract

This article aims to investigate the efficiency of the SP500 indexes over the 1996-2021 period. In our study, we also analysed the possible link between the size effect, the value effect and the business cycles. The results show the existence of a size effect and a reversed value effect during the whole period. However, when considering the sub periods, the anomalies are not persistent. These results indicate the importance of considering the market as a whole when studying efficiency.

Keywords: market risk premium, GARCH, CAPM, anomalies, size effect, Book to Market effect, US stock market, Efficient Market Hypothesis (EMH)

JEL Classification: G1, G10, G12, G14, G17

1. Introduction

One of the pillars of financial theory is the market efficiency hypothesis. It is defined as the reference for the pricing model for financial assets. According to Fama (1965, 1970, 1991) and Jensen (1978), a financial market is efficient if and only if all the information available concerning each financial asset listed on the market is immediately integrated into the price of that asset (e.g. Bachelier, 1900; Samuelson, 1965). However, the concept of the anomaly poses a problem regarding the validity of the Capital Asset Pricing Model (CAPM) and the market efficiency hypothesis. In developed stock markets, anomalies are a well-documented stylized fact. The cross-sectional stock returns are among the most robust findings, two sorts of anomalies exist: the cross-sectional stock anomalies, for example the size effect, value effect (or Book to Market anomaly, PER effect) (Note 1), etc and the seasonal effect, for example the January effect, the week effect, etc. (e.g. the early researches: Graham and Dodd 1934; Basu, 1977; Jensen, 1978, Banz, 1981; Fama & French, 1992; Ibbotson 1997; Reinganum, 1999).

There are two main reasons for this interest. The first reason is theoretical: if it is possible to show that the investment strategy based on anomalies is capable of systematically beating the market, then the theory of market efficiency hypothesis would be faulty. The second reason is practical: if there are investment strategies whose performances are on average higher than that of the indices, investors would have an interest in identifying them. Theoretically, regulations and regulators have rendered the stock market more efficient. In such cases, we expect a mitigation in cross-sectional stock anomalies in the U.S. stock market. However, in the U.S. stock market, those anomalies persist. These observations led us to analyse recent changes to the size effect and the value effect on the American market. We study two of most representative cross-sectional anomalies: Size effect and Value effect. Those two anomalies can be demonstrated not only through cross-sectional samples but also by using the stock market indices (Ibbotson yearbook 1997, 2007, 2016). The predictive ability of Size factor and value factor (book-to-market ratio, PER ratio, etc.) are the signs of inefficiency, investors can use financial products such as ETF or Tracker to make profits.

This paper makes several contributions to the literature. Our study covers the period from 1996 to 2021, this period also coincides with major financial crises: Stock market downturn of 2001-2002 and subprime mortgage crisis 2007-2010, which had a major impact on the world economy. During this period, not much research on the size effect and value effect was published, particularly on the US stock market. Thus, our paper contributes to the existing finance literature by investigating the size effect in the US stock market during the recent period. Fama (1970) stated that returns of financial assets have no memory, the weak form of the Efficient Market Hypothesis (EMH) implies that return rates should not have dependence. One of the most used methodologies to analyse the dependence is the use of the GARCH models. If the EMH is verified, return rates have no memory (a random walk).

The rest of the paper is organized as follows. Section 1 presents the origin and classic explanations of size and value effect. Section 2 develops the methodology, we detail the data. The Section 3 analyses the results, and section 4 concludes.

2. The Size and Value Effect, Origin and Interpretation

Banz (1981) demonstrated that small cap securities generated greater returns and attributed this overperformance to the remuneration of an additional risk factor. The size effect poses a problem with regards to the validity of the Capital Asset Pricing Model (CAPM) of Markowitz (1952), validity according to which the expected yield of securities depends on the systematic risk level (the Beta). Since Banz's publication, empirical studies have revealed that the size effect and Book to market effect have decreased. To illustrate these findings, we can cite the work of Kim & Burnie (2002), L'Her, Masmoudi, & Suret (2002) and Van Dijk (2011). However, not all the empirical studies agree with these findings. Amel-Zadeh (2011) detected the strong size effect in the German stock market and the regularity of return cannot be explained by differences in systematic risk. Jiang & Zhang (2013) show that the anomalies are driven mainly by the samples of stocks: the Size and BM effects are highly significant for Equal-weighted (EW) portfolios, and the VW (Value-weighted) portfolios don't have significant anomalous returns. Xiao (2015) confirms the existence of a positive relationship between the EW measure of idiosyncratic risk and stock returns by using Nikkei 225 index stocks. According to behavioural finance researchers, size effect is proof of the irrationality of individuals. On the other hand, researchers who support the concept of rationality suggest that size effect can be attributed to risk factors other than the market (Zhong et al. 2014, Hou et al. 2020, Darolles et al 2022).

Value effect (or the PER effect, or the Book-to-Market effect) was discovered by Basu (1977). **S**ecurities with low PERs have higher returns than securities with high PERs (Basically, value stocks consistently out-perform growth stocks on average in the world. See, Graham and Dodd, 1934, Basu, 1977, Fama and French, 1992, 1998, Ibbotson 1997, 2007, 2016, Patton and Weller, 2020). In other words, the risk premium linked to securities with high PER is lower than for securities with low PER.

According to Fama and French (1992, 1995): the value premium may be compensation for systematic risks other than market portfolio return. This relationship could be explained as follows: securities with a low PER or Book to market ratio (B/M) are undervalued compared to their real value. On the contrary, securities with a high PER or B/M ratio are overvalued. These overvaluations and undervaluations will gradually correct over time. The prices of securities with a high PER or B/M will therefore fall or, at least, increase more slowly than the market. Conversely, the prices of securities with low PER or low B/M will rise faster than the market. The profitability of securities with high PER will therefore be low, while the profitability of securities with low PER will be high. Miller (1977), Stambaugh et al. (2012), Cheema and Scrimgeour (2019) show that overpricing is more prevalent than underpricing suggesting that anomalies are driven by overpricing. This very logical explanation, however, is in total opposition to the theory of efficiency. Indeed, if the markets are efficient, the securities can neither be undervalued nor overvalued. All information is instantly induced in the course and the market price is at all times exactly equal to the real value of the goods.

According to DeBondt and Thaler (1985) and Lakonishok et al. (1994), the underperformance of growth stocks relative to value stocks is that investors are irrationally exuberant about the prospects of innovative glamour companies. Betermier et al (2017) examine value and growth investments of Swedish residents between 1999 and 2007, they reveal that growth investing is strongly linked to aggregate risk and the investors with high exposure to macroeconomic risk tilt their portfolios away from value. They related that the value tilt to household characteristics: «Value investors are substantially older, are more likely to be female, have higher financial and real estate wealth, and have lower leverage, income risk, and human capital than the average growth investor. By contrast, men, entrepreneurs and educated investors are more likely to invest in growth stocks. »(Betermier et al., 2017, p.6)

To reconcile the size effect and value effect with the CAPM, Fama and French (1993) proposed incorporating additional risk factors into it (The three-factor model), as the beta was no longer the sole source of risk. The three-factor model tries to explain the profitability of stocks in terms of market, level of capitalization and value effect. Jagannathan and Wang (1996) proposed the conditional CAPM. Campbell and Vuolteenaho (2004) mentioned the notion of cash-flow risk. Zhang (2005) introduced the concept of costly reversibility of physical capital. Doron and Tarun (2006) proposed a time-varying beta version of multifactor models to capture the size and book-to-market effects. However, there is still an ongoing debate about whether expected returns are explained by risk factors or by investor behavior. Our study examines the recent evolution of the cross-sectional anomalies, if they attenuated, which implies that the US stock market has become more efficient in recent years.

3. Data and Methodology

Our research questions are as follows: Is there a size effect and a value effect over the period from 1996 to 2021 on the American market? Is there a link between the business cycles and these anomalies? Has the American stock market become more efficient? To answer these questions, we used descriptive statistics, the Sharpe Model as well as the GARCH models.

Our data are constituted from the S&P500 Index, the S&P Small Index, the S&P Large Index, the S&P Value Index and the S&P Growth Index on a daily basis over the period 1996-2021. The S&P Small Index and Large Index seek to measure the small-cap and the large-cap segments of the U.S. equity market. S&P Style Indices divide the complete market capitalization of each parent index into growth and value segments. Constituents are drawn from the S&P 500. S&P value index measures value stocks using three factors: the ratios of book value, earnings, and sales to price. S&P 500 Growth Index measures growth stocks using three factors: sales growth, the ratio of earnings change to price, and momentum (Note 2).

Data on prices spans the period from January 2nd, 1996 to April 26, 2021, a total of 6,374 observations for each index (Figure 1). The requisite data is obtained from the Factset database.

Factset is a financial database and designates a software editor. The company provides financial information and analytical software for investment professionals. The graphical representation of prices is illustrated as Figure 2 and Figure 3, those illustrations compare the development of these fives price indices and indicate the domination of the SP Small price index.

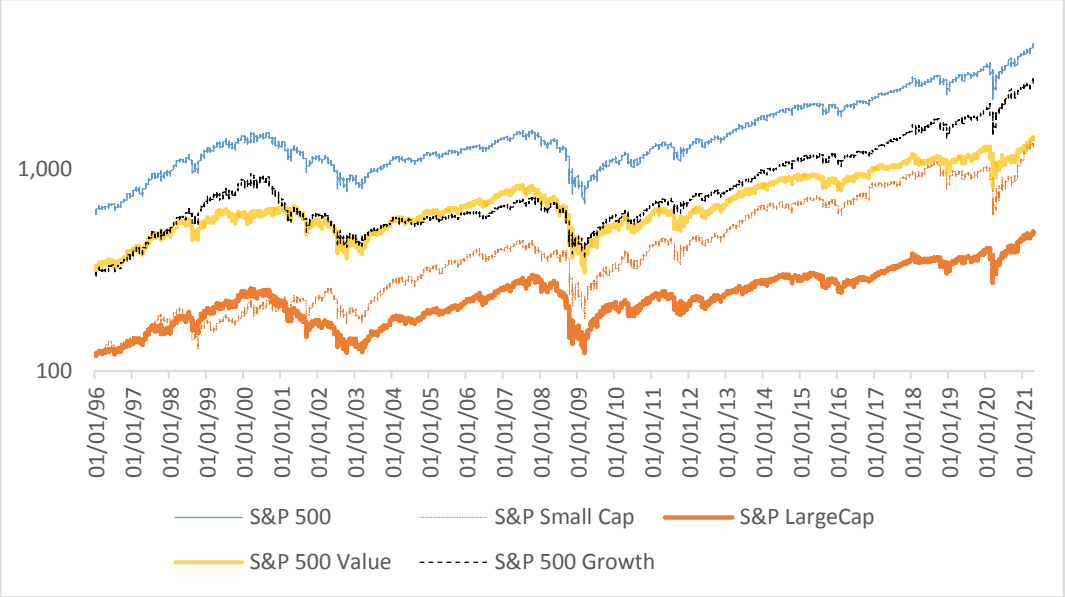

Figure 1. Prices of five Indexes (Logarithmic scale, Daily, 1996 - 2021)

The prices of the financial series can be difficult to model directly. The trend is far from stationary since its mean level is not constant and rises over time. A standard procedure to model the logarithmic price return rather than prices themselves. The logarithm of the gross return R ln is given by:

$$\boldsymbol{R_{ln} = ln(\frac{P_t}{P_{t-1}})} \quad (1)$$

where,

R_t= the rate of return at time t

P_t = the price at time t

P_{t-1} = the price just prior to the time t.

In general, the movements of the stock indices series are non-stationary. The Dickey-Fuller test and the Phillips-Perron test confirm this (Tables 1, 2 in the Appendix). We convert the daily price into the return series, the results show that the series of return are stationary (Tables 3, 4 and 5 in the Appendix).

Table 1. Summary statistics for returns (Daily, 1996 - 2021)

	Obs	Mean	Std. Dev.	Min	Max	Skewness	Kurtosis
SP500	6372	0.0003749	0.012258	-0.119841	0.115800	-0.18529	10.1653
SP Value	6372	0.0003110	0.012537	-0.116314	0.111429	-0.23886	10.1779
SP Growth	6372	0.0004304	0.012692	-0.122634	0.128278	-0.06571	9.17558
SP Large	6372	0.0002710	0.010062	-0.093257	0.092953	-0.41435	9.42675
SP Small	6372	0.0004810	0.014303	-0.133089	0.090067	-0.33599	7.02127

We use the Sharpe model to study risk adjusted returns. Using the Sharpe model with different indexes serves to detect the presence of the size effect and the value effect during the study period. The Sharpe model is a static model that allows us to explain the return of each security according to the return of the market (The SP500 index is used to determine the Market index return).

$$R_{i,t} = a_i + \beta_i RM_t + \varepsilon_{i,t} \quad (2)$$

Where:

$R_{i,t}$ Rate of return of security (i) on date (t);

RM_t Market index return on date (t);

Alphas and betas are constants and specific to each security.

As in most efficiency studies, the choice of the period greatly influences the result of the analysis. On the American market, studies revealed the cyclical nature of stock market anomalies, and some have even hailed the disappearance of these anomalies (e.g. Reinganum, 1999; Horowitz et al., 2000; Kim and Burnie, 2002; L'Her et al., 2002; Schwert, 2003; Van Dijk, 2011; Darolles et al 2022) (Note 3). Fama and French (1993) suggested that the higher than expected returns of value stocks and small caps offset the additional risk inherent in these securities for shareholders. In fact, value stocks and small caps are susceptible to being financially weakened in the event of an economic crisis. Generally speaking, small and value style shares are penalised to a greater extent during times of crisis due to debt and credit problems (e.g Beck et Dermirgue-Kunt, 2006; Arshanapalli, Fabozzi and Nelson, 2006; Switzer and Tang, 2009).

Our study assesses the effects of the GFC (the Lehman Brothers bankruptcy on September 15, 2008, is used as a breakpoint to divide the sample) on efficiency ranking (e.g. Mensi et al.,2017 ; Ferreira et al. ,2018). The effects of the GFC provide evidence for two sub-samples: January 2000 – September 15, 2008, and September 16, 2008 – March 2018. We see a sharp decrease of our fives indices during September 2008, which corresponds to the Lehman Brothers collapse.

Traditionally, the SP500 index is the most used U.S. stock market. This index represents the global economic trend of large U.S. companies. To determine the sub periods, we have chosen to use the SP500 index to delimit bullish periods from bearish periods. During the study period, the SP500 index shows three bullish periods and two bearish periods:

Bull markets:

From January 2, 1996 to September 8, 2000 (Internet bubble, Period of euphoria)

From March 12, 2003 to September 15, 2008 (Real estate bubble, Effects of the GFC)

From March 10, 2009 to April 26, 2021

Bear markets:

From September 11, 2000 to March 11, 2003 (Internet bubble and the September 11 attack)

September 16, 2008 to March 9, 2009 (Subprime crisis)

4. Empirical Findings (Analysis and Results)

The objectives of our research are as follows: we verify the existence of the size effect and the value effect on the US market over the period from 1996 to 2021 using the SP500 indices. In our study, we also analysed the possible link between the size effect, the value effect and the business cycles. We use the Lehman Brothers collapse (September 15, 2008) to divide the whole sample into sub-samples: January 1996 – September 15, 2008, and September 16, 2008 – April 2021.

By using the return series, we obtained 5 price indexes, Table 2 indicates what one dollar invested at the start of 1996 would return at April 2021 for each price index, as well as the geometric monthly return and the volatility of the arithmetic monthly return for each index. We studied the 5 indices during the whole period and during the sub periods, we noted the existence of a size effect and an inverse value effect.

4.1 Evolution of the Size Effect and the Value Effect Over the Whole Period

Preliminary analyses show a size effect during whole period. Table 2 indicates what one dollar invested at the start of 1996 would return at the end of April 2021 in index, as well as the average geometric daily return and the volatility of the arithmetic daily return for each index. The result indicates the domination of the SP Small price index and the SP Growth index during the whole period.

Table 2. Development of the five price indices (Daily, 1996 – 2021)

	End of April 2021 value	Variation	Daily geometric average return	Daily arithmetic average return	Standard deviation of return
SP500	$6,75	574.63%	0.0300%	0.0375%	0.01226
SP Value	$4,39	339.12%	0.0232%	0.0311%	0.01253
SP Growth	$9,28	828.47%	0.0350%	0.0430%	0.01269
SP Large	$4,07	306.84%	0.0220%	0.0271%	0.01006
SP Small	$11,13	1013.37%	0.0378%	0.0481%	0.01430

It can be noted that price indices for small caps register much higher performance levels than large caps, both in terms of returns and volatility. And the SP Growth index registers much higher performance levels than the SP Value index, but the standard deviation levels are not significantly different. In addition, concerning small cap and large cap, as Table 1 and 2 demonstrate, there is a positive relation between the returns and volatility levels. The price index for small caps thus shows a higher return rate and a higher volatility rate than for large caps.

Figure shows the domination of the S&P Small Index, 1 Dollar invested in the S&P Small Index at the start of 1996 would return 11.13 Dollars on 2021 (April 26, 2021). By contrast, 1 Dollar invested in the S&P Large Index would return 4.07 Dollars after 25 years. We can conclude from this that returns from the group of small capitalisation companies is greater than those from the group of large capitalisation companies over the period as a whole.

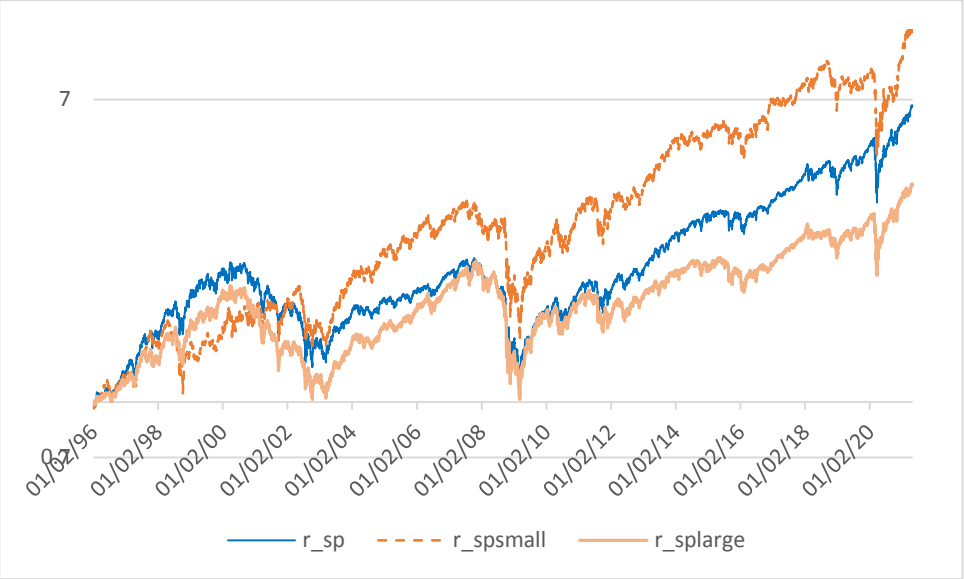

Figure 2. S&P Small Index V.S. S&P Large Index (Logarithmic scale, Daily, 1996 – 2021)

According to the results of the comparison between the S&P Value Index and the S&P Growth Index, we can notice that, after 25 years, 1 Dollar invested in the S&P Value Index would return 4.39 Dollars. At the same time, S&P Growth Index would return 9.28 Dollars. We can conclude from this that returns from the group of Growth companies is greater than those from the group of Value companies over the period as a whole.

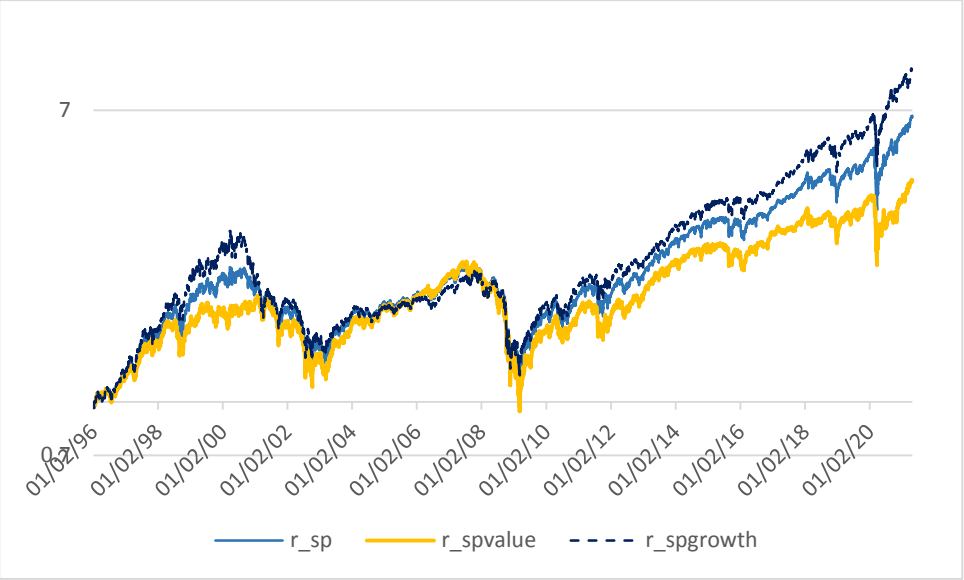

Figure 3. S&P Value Index V.S. S&P Growth Index (Logarithmic scale, Daily, 1996 – 2021)

Table 3 summarises the results of the regressions according to the Sharpe model. Application of the Sharpe model to these different indexes allows us to test the existence of the size effect and the reversed state of the Value effect on the study period as a whole, with excess returns corresponding to the regression alpha, beta is the coefficient of index i, the measure of systematic risk. The regressions were carried out using the ordinary least squares method on each group for the period ranging from 1996 to 2021. For each regression model, the obtained results are presented based

on the SP 500 index as a proxy of the market portfolio.

Table 3. Estimations according to the Sharpe model (Market index: SP 500)

	Alpha	t_α	**Beta**	t_β	**R ²**	**F-stat**
SP Value	-6,02E-05	-1,53328	0,99027	309,04208	0,96823	95507,0087
SP Growth	5,18E-05	1,474090	1,00994	352,66940	0,97533	124375,712
SP Large	-2,62E-06	-0,045394	0,72989	155,07261	0,88914	24047,5167
SP Small	9,98E-05	1,134148	1,01651	141,59275	0,87113	20048,5071

According to our calculations, the alphas are close to zero. This result confirms the size effect. The regression confirms the size effect by using the data of the SP 500 index, the alpha of the SP Small index is greater than in the other indexes. By using the daily return, the alpha of the Small index is 9.98e-05 against that of SP value which is negative at -6.02e-05.

4.2 Anomalies Before and After the GFC

Table below indicates what one dollar invested at the start of each period would return at the end of period for each price index. Amount in bold, the best performance for each investment strategy.

Table 4. Development of the five price indices during sub periods (Daily, 1996 – 2021)

	SP500	SMALL	LARGE	VALUE	GROWTH
Internet bubble	$2,41	$1,82	**$2,03**	$1,94	**$2,90**
Internet bubble crisis	$0,54	**$0,79**	$0,51	**$0,60**	$0,47
Real estate bubble	$1,49	**$2,10**	$1,74	**$1,57**	$1,41
before the GFC	**$1,92**	**$3,01**	**$1,80**	**$1,83**	**$1,93**
Subprime crisis	$0,57	$0,50	**$0,57**	$0,51	**$0,62**
Covid crisis	$0,66	$0,59	**$0,68**	$0.63	**$0.69**
After the GFC	**$6,19**	**$7,47**	**$4,00**	**$4,68**	**$7,77**
Whole period	**$6,75**	**$11,13**	**$4,07**	**$4,39**	**$9,28**

This result proved the existence of anomalies in the period from 1996 to 2021, then to develop our research, we studied the short-term memory properties in our time series. To be able to highlight the relationship between anomalies and the economic cycle, we chose to break down the whole period into two distinct sub-periods. The problem that arises is to precisely determine the short-term memory properties of each of these sub-periods.

Table 5. Estimations according to the Sharpe model before and after the GFC (Market index: SP 500)

	Alpha	t_α	**Beta**	t_β	**R ²**	**F-stat**
Before Subprime crisis						
SP Value	-2,499E-06	-0,04336	0,94964	185,70516	0,95665	34486,4066
SP Growth	-1,189E-06	-0,02198	1,04820	218,328176	0,96807	47667,1925
SP Large	3,480E-05	0,46281	0,70942	106,350596	0,88299	11310,4493
SP Small	0,000179	1,51525	0,88858	84,7742522	0,83197	7186,67384
After Subprime crisis						
SP Value	-0,000121	-2,33292	1,02027	257,305787	0,97687	66206,2679

SP Growth	0,0001084	2,49240	0,98167	297,461969	0,98254	88483,623
SP Large	-4,211E-05	-0,48115	0,74501	112,230198	0,89377	12595,6174
SP Small	8,981E-06	0,07117	1,11089	116,076361	0,89968	13473,7217

An efficient market corresponds to rational investor and rational governors. The size effect and the reversed value effect mean the S&P indexes return series are not efficient, to confirm our intuition, we test for efficiency of S&P indexes using GARCH models. Table 6 presents the results of the models fitted to the data on returns. The outputs of GARCH (1,1) on the returns show that the alphas are not statistically significants. The variance equation illustrates that all the terms are statistically significant at 1% level of significance. It means that there is the short-term memory during the whole period and a certain inefficiency in the SP indexes.

Table 6. Estimated coefficients for the GARCH models

		Whole period	Before the GFC	After the GFC			Whole period	Before the GFC	After the GFC
SMALL	Alpha	.0001229	.0001773	-.0000746	VALUE	Alpha	-.0000957	-.0000343	-.0001378
		0.102	0.118	0.460			0.000	0.366	0.000
	Beta	.9942537	.8889475	1.150462		Beta	.9958174	.9692008	1.012886
		0.000	0.000	0.000			0.000	0.000	0.000
	Cons_	3.43e-06	-6.24e-07	3.05e-06		Cons_	-1.48e-07	-1.09e-06	4.69e-07
		0.037	0.887	0.037			0.073	0.000	0.000
	ARCH(L1)	.2448216	.1255888	.2997109		ARCH(L1)	.4037924	.3624851	.4163229
		0.000	0.000	0.000			0.000	0.000	0.000
	GARCH(L1)	.6833521	.8809295	.6448424		GARCH(L1)	.6373148	.777343	.535376
		0.000	0.000	0.000			0.000	0.000	0.000
	Log likelihood	22882.32	11536.75	11445.26		Log likelihood	28558.09	14159.9	14443.5
LARGE	Alpha	.0000166	.000075	-.0000815	GROWTH	Alpha	.0000897	.0000367	.000125
		0.727	0.266	0.205			0.000	0.331	0.000
	Beta	.7315682	.7122043	.7540339		Beta	1.003713	1.029406	.9879998
		0.000	0.000	0.000			0.000	0.000	0.000
	Cons_	-1.75e-06	-3.66e-06	-3.87e-08		Cons_	-2.43e-07	-1.09e-06	3.50e-07
		0.001	0.002	0.940			0.002	0.000	0.001
	ARCH(L1)	.2193785	.1308379	.3198948		ARCH(L1)	.3579196	.3316115	.3485721
		0.000	0.000	0.000			0.000	0.000	0.000
	GARCH(L1)	.8518331	1.06289	.6735204		GARCH(L1)	.6898927	.8153691	.5848835
		0.000	0.000	0.000			0.000	0.000	0.000
	Log likelihood	25838.01	13031.05	12825.57		Log likelihood	29166.49	14308.89	14934.64

5. Discussion and Conclusion

The sample period of our study covers several episodes of wide instabilities and crises: Stock market downturn of 2001-2002, the 2001 US terrorist attacks, the 2003 Gulf wars, the food price surge of 2007-2008, the subprime mortgage crisis 2007-2010, 2008-2009 GFC, the 2009-2012 Eurozone debt crisis and the COVID crisis. In this paper, we examine the daily data from the U.S. stock market by using S&P indexes, over the 1996-2021 period.

At first, we studied the 5 indices by using return series, we noted the existence of a size effect and an inverse value effect during the whole period. Then, we studied the efficiency of these SP indices using the GARCH models, these 5 SP indices are supposed to represent the trend of the American stock market. The results we obtained then show that the SP500 index tends towards efficiency, we also find that the SP Large index and the SP Growth index tend towards efficiency, but that they represent a form of short-memory. Regarding the short-memory of the SP Small index, which signifies a form of inefficiency. This finding confirms the persistence of size effect: investors demand a high-risk premium for small sized companies. The inversion of the value effect means that the indices S&P Value and Growth are rather efficient. Indeed, over our period of study, growth stocks dominate value stocks, so we see a

phenomenon of mean reversion.

This paper contributes to the current literature by examining the size effect and the value effect in the American stock market. The results have important implications for analyzing the recent evolution on the degree of market efficiency. However, some limitations and future avenues of research exist.

An anomaly during certain periods does not necessarily imply a market inefficiency. For that, we have to show that investment strategy based on seasonality is capable of systematically beating the market. In this article, we focus on the short-memory properties on the SP500 index. The results of studies on the efficiency are conflicting and confusing, results vary depending on the type of data used (cross-sectional data or time series data). The choice and duration of the study period also influence the result (Dury and Xiao, 2022). Bertrand et al. (2020) studied the efficiency of the Chinese stock markets by employing the Hurst exponent analysis, the evolution of the Hurst index indicate that Shenzhen stock market and Shanghai stock market were becoming more and more efficient after the reform. For further analysis of market efficiency, we think we have to complete our study by analyzing the long-term memory properties in our time series.

One of the explanations of the market anomalies that we have not mentioned in our article is the investor confidence, for example, during the real estate bubble, the market was suspicious regarding the growth style stocks, the return was under the domination of value stocks. After the sub-prime crisis, the market preferred prudence, investors favoured growth rather than value stocks and they ask for a risk premium for small-sized stocks. Boussaidi and Dridi (2020) found an evidence that the risk cannot explain the momentum effect, Zakamulin and Giner (2022) develop a tractable theoretical model that contributes to the understanding of the trend-following strategy's risk. For our further research it would be interesting to associate the transverse anomalies (the size effect and the value effect) with the momentum effect.

Traditionally, small caps stocks and value style stocks are considered to contain more cyclicals and large caps stocks and growth style stocks are considered more defensive. Darolles et al (2022) find that the upward (downward) size state is characterized by strongly size spreads, and is correlated with the lagged changes of the composite leading indicator. As an avenue for future research, we would complete our analysis of performance by size factor or value factor, we could identify and analyse the different drivers of performance, including cyclical and defensive sectors.

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Notes

Note 1. In this article, for practical reasons, we use the term Value factor to designate this risk factor identified by Basu (1977) (e.g. Fama & French, 1992 and Reinganum, 1999), sometimes known as the Book-to-Market effect or PER effect.

Note 2. Source: https://www.spglobal.com/spdji/en/

Note 3. Reinganum (1999) suggested that the size effect could be predicted and that large companies outperformed small companies during periods of unfavourable economic conditions. Horowitz, Loughran, & Savin (2000) observed that the size effect had disappeared during the period encompassing 1981 to 1997. Schwert (2003) suggested that the size effect had disappeared between 1982 and 2002. Van Dijk (2011) suggested that the phenomenon had been cyclic in the period between 1927 and 2005. Kim & Burnie (2002) advanced the hypothesis according to which size effect might be driven by the economic cycle. L'Her, Masmoudi and Suret (2002) also pointed out that risk premiums vary according to economic conditions.

Appendix

Table 1. Dickey-Fuller test for SP500

Test statistic	1% critical value	5% critical value	10% critical value	p-value for Z(t)
0,238	-3.430	-2,860	-2,57	0.9743

Table 2. Phillips-Perron test for SP500

	Test statistic	1% critical value	5% critical value	10% critical value
Z(rho)	1,115	-20.700	-14.100	-11.300
Z(t)	0,548	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.9863

Table 3. Dickey-Fuller test for SP500 return

Test statistic	1% critical value	5% critical value	10% critical value	p-value for Z(t)
-73,81	-3.430	-2,860	-2,57	0.9743

Table 4. Phillips-Perron test for SP500 return

	Test statistic	1% critical value	5% critical value	10% critical value
Z(rho)	-4756,569	-20.700	-14.100	-11.300
Z(t)	-74,371	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

Table 5. Dickey-Fuller test for returns

	SP500	SMALL	LARGE	VALUE	GROWTH
Whole period	-84.303	-80.446	-69.647	-83.209	-83.943
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Before the GFC	-53.937	-50.811	-46.087	-53.143	-54.139
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
After the GFC	-64.010	-60.372	-51.415	-62.766	-64.148
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

1% critical value: -3.430, 5% critical value: -2.860, 10% critical value: -2.570

p-values are given in parentheses.