Segmentation in Consumer Durables Markets

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Abstract

Consumer durables markets are often observed to be segmented, with some firms producing highly reliable output and offering good warranty deals, while others produce less reliable output and offer less attractive warranties, but charge a lower price. The model of this paper defines reliability as the objective probability of product failure, not as a characteristic of individual goods. Reliability, thus defined, is treated as a choice variable of the firm. This approach to reliability is incorporated into a duopoly model which explains the phenomenon of segmentation described above.

Keywords: consumer durables, reliability

1. Introduction

In consumer durables markets, firms are frequently observed to specialise their production and marketing operations so as to occupy definite segments of the market. Some may offer highly reliable products to the market, perhaps along with valuable warranty arrangements. Other firms may opt for low reliability output, perhaps accompanied by less valuable warranties. The former will typically charge higher prices than the latter. Barber and Darrough (1996) study the international market for non-luxury cars. Japanese firms market expensive, highly reliable cars with good warranty deals while American (or perhaps Italian?) firms specialise in less reliable, less expensive cars with less valuable warranties. Production and quality control technologies are more or less common knowledge around the world, so this form of specialisation, and the consequent international intra-industry trade in cars, requires some explanation other than technological differences. One might appeal to different factor endowments, but for mass produced products such as non-luxury cars this seems an implausible approach.

Epple and Raviv (1979) and Saving (1982) argue that product reliability may be independent of market structure. Goering and Read (1995) develop a two-period oligopoly model in which they establish that this independence result is true only under very limited conditions, though it generally holds if warranties are legally compelled. Their results provide a partial explanation for Avinger's (1981) empirical findings on product obsolescence in the vacuum tube and electric lamp industries.

A possible theoretical approach to this issue can be found in the product differentiation literature. Models with vertically differentiated products have been used to explain intra-industry trade under a number of different market structures. For example, Falvey (1981) develops such a model while retaining significant aspects of the Heckscher-Ohlin-Samuelson framework. His approach is to assume that relative factor intensity is the main force behind vertical product differentiation. The predicted pattern of trade is consistent with the basic Heckscher-Ohlin-Samuelson result, i.e. each country exports the quality which uses its relatively abundant factor intensively. Shaked and Sutton (1984) analyse vertically differentiated production under oligopoly. They develop a three-stage model: firms decide on entry in the first stage, quality in the second stage and price in the third stage. Thus the number of firms is endogenous. Intra-industry trade can easily emerge in such models but its pattern is dependent upon the distribution of income across trading partners. See also Vandenbosch and Weinberg (1995) and Lambertini (1997). Choi and Shin (1992) develop a vertical differentiation duopoly model in which all the subgame perfect equilibria in pure strategies are asymmetric (one firm setting a high price and high quality and the other a low price and low quality). Deneckere and de Palma (1998) develop a model of a vertically differentiated durable goods duopoly. In the version of their model with endogenous quality choice it is difficult for the low quality firm to soften

competition by lowering the quality of its product. This leads to less vertical differentiation than would arise in a market for nondurable goods.

Gowrisankaran and Rysman (2012) focus on consumer preferences for new durable goods. Their model is dynamic, entailing repeat purchases, rational expectations and persistently heterogeneous consumer tastes. They estimate their model using panel data from the US camcorder industry. A similar approach is used by Berry et. al. (1999) to analyse trade policy and by Schiraldi (2011) to analyse automobile subsidies.

The model of this paper is in a similar vein to the product differentiation literature and establishes the existence of an asymmetric equilibrium in pure strategies. However there are some crucially different features. Firstly it offers a definition of "reliability" which is sharply distinguished from the notion of "quality" in the existing literature. On this new definition, reliability can be thought of as a relatively flexible variable (similar to price or quantity) and there is consequently no need for a multi-stage game framework. Secondly it introduces two-dimensions of product differentiation, which cannot, therefore, strictly be described as "vertical". The first dimension is the reliability of the firms' output: the second dimension is the warranty payment which firms can choose to offer to the market. These two variables (reliability and warranties) cannot, in equilibrium, be chosen arbitrarily by firms, because they are linked by consumers' behaviour towards risk.

2. Modelling Reliability

In the model presented in this paper "reliability" will be defined as "the probability of a consumer durable not breaking down within some given time period". The probability referred to here is an objective frequency. For example, if a car manufacturer produces 100,000 cars each year and 93,000 do not break down within a given time period (say one year) the reliability of these cars is 0.93. It will be assumed that the firm chooses reliability (as defined here) by varying the stringency of its quality control procedures. Thus the firm knows for sure that 7,000 of its cars will break down within the year, but it neither knows nor cares *which* 7,000 they will be. Now suppose the firm offers a one-year warranty with its cars, promising compensation in the event of a breakdown. In this model the firm faces no uncertainty, it knows its revenue and production costs: it knows that there will be 7,000 claims under the warranty (though not which customers will make them) and it knows how much it will have to pay out per claim (that can either be treated as endogenous or imposed by a regulator). There is therefore no uncertainty about its profits.

Warranties, whether voluntary or legally compelled, have an important bearing on decisions affecting reliability because the higher the reliability of a firm's marketed output, the lower the expected warranty costs experienced by the firm (*ceteris paribus*). This connection between warranties and reliability has been apparent to managers for some time. Wright (1980), for example, describes events at General Motors:

"I instituted a programme for testing and repairing faulty cars as they came off the assembly line - and the results were phenomenal. It cost about $8 a car, which drove The Fourteenth Floor up the wall. But I figured one way or the other we would end up fixing the defects or paying to have them fixed through recall campaigns or dealer warranty bills........ The internal quality control audit revealed a 66% improvement in the quality of a Chevrolet coming off the assembly line between 1969 and 1973 models. And most important, warranty costs of our new cars were down substantially."

It will be assumed that consumers have no knowledge about individual products but do know the reliability of each firm's output (in the sense defined here). Consumers read *Which?* magazine or *Consumer Reports* or obtain information on reliability from other sources. Firms will be assumed to have the same (limited) information. This is a plausible assumption because it is usually impossible or extremely costly for firms to obtain information on each example of its output before it is sold. Firms will be assumed to vary reliability (as defined here), by varying the stringency of its quality control procedures. It will be assumed that higher reliability entails higher costs (e.g. rework or scrapping costs). Thus, the car manufacturer will be able to reduce (or increase) the number of breakdowns in a given time period without knowing (or caring) which vehicles will break down and which will not. It will therefore be assumed to know the reliability of its output (as defined here), without knowing individual products will break down.

In this model the firm faces no uncertainty, though this is not true of consumers, who are assumed to be risk averse. In consumer durables markets, each consumer typically owns one example of the good, and is thus extremely concerned at the prospect of its breaking down. The firm, by contrast, supplies many examples of the good, and may well find it profitable to operate a risk-pooling warranty scheme. Under these assumptions there arises a demand, on the part of consumers, for insurance. This might, as mentioned above, be provided in the form of a product warranty

offered by the firm, or an insurance policy provided jointly with the product. Throughout the paper attention will be confined to *voluntarily* offered warranties or compensation, though the model is readily modified to include legally compelled compensation. It could also be modified to cover different degrees of product breakdown, or product hazard and safety issues.

In the model of this paper, consumers' preferences have three distinct aspects.

- The consumer's preference for the good *in its un-broken-down state*. This varies across consumers and is exogenous.

- The consumer's degree of risk aversion. For convenience this is assumed constant across consumers and is exogenous.

- The probability of the good not breaking down within some given time period (i.e. the *reliability* of the good). Subjective and objective probabilities are, by definition, identical in this model. It is an essential feature of the model that this probability is endogenous (determined by firms' decisions) and the same for all consumers.

The details of consumers' utility functions are developed in section 3 of the paper.

Note that the model developed in this paper differs sharply from that presented in the literature on product quality. "Quality" is usually taken to be a characteristic of goods which is such that more is preferred to less, *ceteris paribus*, by all consumers. It is typically assumed that all examples of a good produced by a given firm are of the same quality, and that the firm is able to vary this quality (and in so doing to vary its costs). This kind of quality variation could come about, for example, by redesigning the product or adopting a different technique of production. Quality is therefore typically thought of as a relatively inflexible variable (compared with quantity or price) and is usually assumed to be set in an early period in a multi-stage game model. Reliability, as defined here, is rather different. It is a feature, not of individual goods, but of the *distribution* of goods produced by a particular firm, namely the *objective* probability of breakdown not occurring. In contrast to quality, reliability, as defined here, is a relatively flexible variable. It can be varied by changing the stringency of quality control procedures, though an increase in reliability brought about in this way would entail higher costs, such as rework or scrapping costs.

The literature on experience goods focuses on *asymmetric* information. "Nature" dictates all relevant characteristics of each good to the supplier before sale, but these are unknown to the consumer at that stage. (E.g. the car's gear box will fall out in the first year.) The supplier's problem is thus one of *signalling*. Perhaps by means of advertising, or offering a warranty or compensation deal, the supplier of high quality goods seeks to signal his high quality to consumers in a credible way. See, for example, Grossman (1981), Milgrom and Roberts (1982, 1986), Kreps and Wilson (1982), Klein and Leffler (1981), Shapiro (1983), and McClure and Spector (1991) for models of this type. Signalling models are of use in analysing Akerloff's (1970) famous lemon seller: he has one car to sell, which he has owned for long enough to know all its idiosyncrasies. Potential buyers are ignorant of these and an enforceable warranty is impossible; thus the problem really is one of signalling. Such models do not describe well the new car market, where:

1. Car firms have thousands of cars for sale but most buyers want a maximum of one

2. Consumers read road tests and are well informed about the reliability (in the sense defined above) of each firm's cars

3. Car firms provide warranties with all their vehicles

4. "Nature" does not dictate reliability to firms; they can vary this by varying the stringency of their quality control procedures

A standard problem, often assumed away in the literature, is that of moral hazard on the part of consumers. If consumers can themselves influence the probability or size of a claim under the warranty, for example by failing to take proper care of the good during consumption, then the economic role of warranties may be reduced. See, for example McKean (1970), Oi (1973) and Priest (1981). Goering (1997) discusses the problem of moral hazard facing a durable goods monopolist. For simplicity moral hazard will be assumed away in this paper.

It should be noted that the model presented here focuses on reliability and warranties, deliberately suppressing some other aspects of consumer durables markets. For example, it is essentially a static model, and is not intended to deal with the issue of dynamic consistency in these markets. Moreover, it is a model of symmetric information. In such a model nothing can be gained by admitting the possibility of repeat purchasing, since neither side of the market can learn anything useful about the other.

3. Consumers and Firms

We consider a consumer durables duopoly (firms indexed by i=1,2) in which consumers can choose to purchase one unit from firm 1 or one unit from firm 2 or nothing. Let z represent the number of consumers choosing to purchase their one unit. For mathematical convenience take z to be a strictly positive real variable. Each consumer has a money budget M and pays a price p_i to firm i (i=1,2). If the consumer durable does not break down within the given warranty period the z'th consumer receives a stream of services which she values at $f(z)$. Note that $z > 0$ and $f'(z) < 0$. If the good does break down within the warranty period the consumer values the stream of services at zero, but the firm makes a voluntary warranty payment of $\beta_i (i = 1,2)$ to her. Costs of writing and enforcing the warranty are ignored. Thus the z'th consumer receives income stream:

$$x = M - p + f(z) \tag{1}$$

if the good does not break down, and

$$y = M - p + \beta \tag{2}$$

if it does.

The reliability of output will be defined as in section 2, as the probability (R) of the good not breaking down. Consumers are assumed to be risk-averse maximisers of expected utility. Throughout the paper it will be assumed (following the discussion of section 2) that consumers are well informed about reliability and hence that their subjective probability of the good not breaking down is equal to the objective probability (R).

The z'th consumer maximises expected utility:

$$V = R.U(M - p + f(z)) + (1 - R).U(M - p + \beta) \tag{3}$$

Note that the model is one of partially endogenous preferences, since R is an endogenous variable. Clearly $U'(.) > 0$, and, to ensure risk aversion, it is assumed that $U''(.) < 0$ (i.e. the function $U(.)$ is assumed concave).

Reliability costs are discussed at some length in the management literature (e.g. see Bowbrick, 1992). Groocock (1986, p53) points out:

> "Because the products might be defective they must be inspected and tested. This results in appraisal costs.....Products may also fail a test or inspection, or may fail in the hands of customers. Failure costs are then incurred.......(since the firm) must rework or replace the failed product during manufacturing, or replace or repair the product for customers, for example, under warranty."

The model developed here formalises these costs by assuming that production costs are increasing in the reliability (R) of output, and by incorporating warranty costs into the firm's profit-maximising decision. Average and marginal production costs, at a given reliability level, will be assumed constant. Note that $z_j (j = 1,2)$ is the output of the j'th. firm, $R_j (j = 1,2)$ is the reliability of the j'th. firm's output, $\beta_j (j = 1,2)$ is the warranty payment offered by the j'th. firm and $p_j (j = 1,2)$ is the price charged by the j'th. firm.

Adopting the assumptions set out above a suitable production cost function is:

$$z_j.C(R_j), \text{ where } C'(R_j) > 0 \text{ and } C''(R_j) > 0 \text{ for } 0 < R_j < 1. \tag{5}$$

Note that both firms are assumed to have identical production and quality control technologies, and therefore the same $C(.)$ function. The number of times that the product breaks down is clearly $z_j.(1 - R_j)$, and thus warranty costs are given by:

$$\beta_j z_j (1 - R_j) \tag{6}$$

Thus each firm maximises profit, given by:

$$\Phi_j = p_j.z_j - z_j.C(R_j) - \beta_j z_j (1 - R_j) \tag{7}$$

4. Structure of the Duopoly Model

Following the discussion of section 2, reliability is treated as a relatively flexible variable, in contrast to "quality" as defined in the standard literature. It can therefore be thought of as being chosen simultaneously with warranties and prices. There is therefore no need to appeal to a multi-period game approach. In fact the equilibrium concept adopted here is "augmented Bertrand equilibrium"; i.e. Nash equilibrium in price, warranty and reliability. For simplicity the entry decision is not modelled. By assumption, consumers can purchase one unit from firm 1 or one unit from firm 2, or no units: they cannot purchase more than one unit.

Let $V_i(z)$ ($i = 1,2$) be the expected utility obtained by consumer z if she purchases from firm i. I.e.

$$V_i(z) = R_i.U(M - p_i + f(z)) + (1 - R_i).U(M - p_i + \beta_i). \tag{8}$$

Suppose consumers in the interval $[0, z_1]$ are allocated to firm 1 and consumers in the interval $(z_1, z_1 + z_2]$ are allocated to firm 2 (for some z_1 and z_2). Can an allocation of this form be an equilibrium? It will be an equilibrium if it simultaneously satisfies the following two conditions:

A. The "voluntary participation constraint" for each firm: $V_i(z) \geq U(M)$ ($i = 1,2$), for $z \in [0, z_1 + z_2]$.

B. The "single crossing property": $V_1(z) \geq V_2(z)$ for $z \in [0, z_1]$ and $V_2(z) \geq V_1(z)$ for $z \in [z_1, z_1 + z_2]$.

Condition A ensures that consumers prefer to consume the output rather than not consume it. Condition B guarantees that consumers purchase from the firm whose output yields the higher expected utility. Note that the expected utility functions are monotonically decreasing in z (i.e. $V_i'(z) < 0$, for $i = 1,2$) because $f'(z) < 0$. Hence conditions A and B are as depicted in Figure 1.
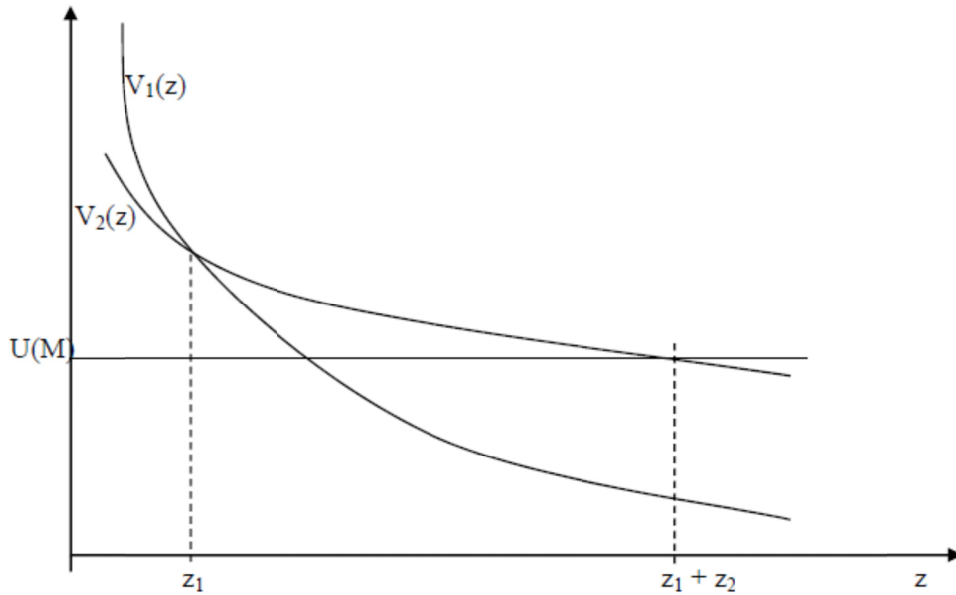

Figure 1

The voluntary participation constraint (condition A) will be assumed to be satisfied by firm 1. This simply amounts to taking the consumers' money budget (*M*) to be low enough. It remains to prove that firm 2 satisfies the voluntary participation constraint, to establish the single crossing property (condition B) and to describe the equilibrium. To achieve these objectives we focus on the first order conditions for each firm. Since the problem is a concave one, these conditions are necessary and sufficient.

Taking λ as Lagrange multiplier, the appropriate Lagrangian for firm 1 is:

$$L = p_1 z_1 - z_1 C(R_1) - \beta_1 z_1 (1 - R_1) + \lambda \left[V_1(z_1) - V_2(z_1) \right] \tag{9}$$

Taking μ as Lagrange multiplier, the appropriate Lagrangian for firm 2 is:

$$M = p_2 z_2 - z_2 C(R_2) - \beta_2 z_2 (1 - R_2) + \mu \left[V_2(z_1 + z_2) - U(M) \right] \tag{10}$$

In the Appendix the first order conditions of these Lagrangians are derived. They, in turn, provide a basis for describing the equilibrium, establishing the single crossing property and showing that the voluntary participation constraint holds for firm 2.

Firstly it is easy to establish that consumer z_1 is indifferent between the two firms (Appendix, Proposition 1) and that the voluntary participation constraint holds for firm 2 (Appendix, Proposition 2). Moreover, the marginal consumers of each firm are fully insured (Appendix, Lemmas 1 and 2). Comparing the two firms in duopoly equilibrium yields the following results:

- Firm 1's output is more reliable than that of firm 2 (Appendix, Proposition 4).

- Firm 1 offers a higher warranty payment than firm 2 (Appendix, Proposition 3).

- Firm 1 charges a higher price than firm 2 (Appendix, Proposition 5).

It remains to establish the single crossing property. This is done in the Appendix, Proposition 6.

5. Conclusions

In consumer durable markets firms are often observed to specialise their production, with some firms producing highly reliable output and offering good warranty deals, while others produce less reliable output and offer less attractive warranties, but charge a lower price. The model developed here offers an explanation of this phenomenon. It develops a new definition of reliability as a relatively flexible variable determined endogenously by firms, and entailing partially endogenous consumer preferences. The model embodies a plausible cost and information structure and introduces two dimensions of product differentiation (reliability and warranties) which, in equilibrium, are linked by consumers' behaviour towards risk.

It is shown that, even when firms have the same cost functions and face the same demand conditions, there exists an asymmetric equilibrium in pure strategies in which firms specialise in the manner described above. The model provides an explanation of the observed pattern of specialisation in the international car market, an explanation more plausible than those based on technology differences or factor abundance.

The model does not deal with the entry decision of firms, treating their number as given, and it assumes away moral hazard. However it could readily be modified to incorporate more than one undesired event, and compulsory as well as voluntary warranties, or to cover product hazard and safety issues.

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Appendix

This Appendix contains the results discussed in the main text. The approach is to derive and utilise the first-order conditions of the Lagrangians given in the main text.

First define:

$$x_1 \equiv M - p_1 + f(z_1) \;\; \text{and} \;\; y_1 \equiv M - p_1 + \beta_1 \tag{A1}$$

The Lagrangian for firm 1 (equation 9 in the main text) can now be written:

$$
\begin{aligned}
L = {} & p_1 z_1 - z_1 C(R_1) - \beta_1 z_1 (1 - R_1) + \\
& \lambda \big[R_1 U(x_1) + (1 - R_1) U(y_1) - R_2 U(M - p_2 + f(z_1)) - (1 - R_2) U(M - p_2 + \beta_2) \big]
\end{aligned} \tag{A2}
$$

We now derive the first-order conditions of this Lagrangian:

$$L_{p_1} = z_1 + \lambda \big[- R_1 U'(x_1) - (1 - R_1) U'(y_1) \big] = 0 \tag{A3}$$

$$L_{\beta_1} = -z_1 (1 - R_1) + \lambda (1 - R_1) U'(y_1) = 0 \tag{A4}$$

$$L_{R_1} = -z_1 C'(R_1) + \beta_1 z_1 + \lambda \big[U(x_1) - U(y_1) \big] = 0 \tag{A5}$$

First we establish:

Proposition 1. Consumer z_1 is indifferent between firms 1 and 2. I.e. the constraint in the Lagrangian of equation 9 (or A2) holds with equality.

Proof. Since $z_1 > 0$ by assumption, it follows from (A3) that $\lambda > 0$. Hence, by complementary slackness, the relevant constraint must hold with equality.

It is now straightforward to establish the useful:

Lemma 1. The marginal consumer of firm 1's output is fully insured. I.e. $y_1 = x_1$, or equivalently, $\beta_1 = f(z_1)$.

Proof. Equations A3 and A4 yield $\lambda R_1 \left[U'(y_1) - U'(x_1) \right] = 0$. Since $R_1 > 0$, by assumption, and $\lambda > 0$ (by Proof

of Proposition 1) it follows that $\left[U'(y_1) - U'(x_1) \right] = 0$. But, by the assumption of risk averse consumers, $U''(.)$

< 0. Hence the function $U'(.)$ is invertible. It follows that. $y_1 = x_1$, or equivalently, $\beta_1 = f(z_1)$, as required.

Corollary 1. $\beta_1 = C'(R_1)$.

Proof. Follows from equation (A5)

Now define:

$$x_2 \equiv M - p_2 + f(z_1 + z_2) \quad \text{and} \quad y_2 \equiv M - p_2 + \beta_2 \tag{A6}$$

The Lagrangian for firm 2 (equation 10 in the main text) can now be written:

$$M = p_2 z_2 - z_2 C(R_2) - \beta_2 z_2 (1 - R_2) + \mu \left[R_2 U(x_2) + (1 - R_2) U(y_2) - U(M) \right] \tag{A7}$$

We now obtain the first order conditions of this Lagrangian:

$$M_{p_2} = z_2 + \mu \left[-R_2 U'(x_2) - (1 - R_2) U'(y_2) \right] = 0 \tag{A8}$$

$$M_{\beta_2} = -z_2 (1 - R_2) + \mu (1 - R_2) U'(y_2) = 0 \tag{A9}$$

$$M_{R_2} = -z_2 C'(R_2) + \beta_2 z_2 + \mu \left[U(x_2) - U(y_2) \right] \tag{A10}$$

It is now straightforward to establish:

Proposition 2. The voluntary participation constraint holds for firm 2. I.e. the constraint in the Lagrangian of equation 10 (or A7) holds with equality.

Proof. Equation (A8) yields $\mu > 0$, since $z_2 > 0$ by assumption. Hence, by complementary slackness, the corresponding constraint must hold with equality, as required.

It is now straightforward to establish the useful:

Lemma 2. The marginal consumer of firm 2's output is fully insured. I.e. $y_2 = x_2$, or equivalently, $\beta_2 = f(z_1 + z_2)$.

Proof. Equation (A9) yields: $z_2 = \mu U'(y_2)$. Substituting in (A8) gives:

$\mu R_2 \left[U'(y_2) - U'(x_2) \right] = 0$. But $R_2 > 0$, by assumption, and $\mu > 0$, from the proof of Proposition 2. Hence:

$\left[U'(y_2) - U'(x_2) \right] = 0$. By the argument of Lemma 1 the function $U'(.)$ is invertible. It follows that. $y_2 = x_2$, or

equivalently, $\beta_2 = f(z_1 + z_2)$, as required.

Corollary 1. $\beta_2 = C'(R_2)$

Proof. Follows from equation (A10)

It is now straightforward to compare the two firms in duopoly equilibrium. First we establish:

Proposition 3. Firm 1 offers a higher warranty payment than firm 2. I.e. $\beta_1 > \beta_2$.

Proof. We have $\beta_1 = f(z_1)$ from Lemma 1 and $\beta_2 = f(z_1 + z_2)$ from Lemma 2. Since $f'(.) < 0$ by definition, the result follows.

Next we demonstrate:

Proposition 4. Firm 1's output is more reliable than that of firm 2. (i.e. $R_1 > R_2$).

Proof. From Lemma 1, Corollary 1 we have: $\beta_1 = C'(R_1)$ and from Lemma 2, Corollary1 we have

$\beta_2 = C'(R_2)$. But $C''(.) > 0$ by assumption. Hence the result follows from Proposition 3.

Finally we establish:

Proposition 5. Firm 1 sets a higher price than firm 2. I.e. $p_1 > p_2$.

Proof. From Proposition 3 we have

$$V_1(z_1) = V_2(z_1) \tag{A11}$$

From Lemma 1 we can substitute $\beta_1 = f(z_1)$ into (A11). This yields:

$$U(M - p_1 + f(z_1)) = R_2 U(M - p_2 + f(z_1)) + (1 - R_2)U(M - p_2 + f(z_1 + z_2))$$

But $f(z_1 + z_2) < f(z_1)$. Noting that $U'(.) > 0$ and that $0 < R_2 < 1$, it follows that:

$$U(M - p_1 + f(z_1)) < U(M - p_2 + f(z_1))$$

Again noting that $U'(.) > 0$, it follows that $p_1 > p_2$, as required.

We now establish the single crossing property (property B in section 5 of the main text).

Proposition 6. The single crossing property holds.

Proof. First define:

$$W(z) \equiv V_1(z) - V_2(z) \tag{A12}$$

Expanding the function $W(.)$ gives:

$$W(z) = [R_1.U(M - p_1 + f(z)) + (1 - R_1).U(M - p_1 + \beta_1)] -$$
$$[R_2.U(M - p_2 + f(z)) + (1 - R_2).U(M - p_2 + \beta_2)] \tag{A13}$$

Now $f(z)$ is monotonically strictly decreasing and $R_1 > R_2$ (Proposition 4). Hence, provided $f(z)$ is steep enough, it must be the case that $W(z) > 0$, for low enough z and $W(z) < 0$, for high enough z. It now remains to establish that $W(z)$ is strictly monotonically decreasing, since that would imply the existence of a unique z_1 such that $W(z_1) = 0$, the required result. Differentiating (A13) yields:

$$W'(z) = f'(z)\left[R_1 U'(M - p_1 + f(z)) - R_2 U'(M - p_2 + f(z))\right] \tag{A14}$$

But $f'(z) < 0$ and $U'(.) > 0$ by assumption. Moreover, $p_1 > p_2$ by Proposition 5 and $R_1 > R_2$ by Proposition 4. Hence the term in square brackets in (A14) must be strictly positive. Hence $W'(z) < 0$, and the result follows.