ORIGINAL RESEARCH

A weighted fuzzy inference method

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ABSTRACT

Ordered weighted averaging operator is introduced to the fuzzy inference for giving suitable weights to the weighted fuzzy production rules. This paper introduces a given weights method of ordered weighted averaging operator based on combinatorial number, according to which we propose an inference algorithm for the weighted fuzzy production rules of weighted fuzzy set. In the process of using this algorithm, a calculation method of weighted fuzzy matching function value and comprehensive similarity measure based on the operator are introduced for calculating the matching degree of the input facts and antecedent portion of the rules reasonability. Example analysis illustrates the feasibility and effectiveness of the given weighted fuzzy inference algorithm.

Key Words: Weighted fuzzy inference, Similarity measure, Ordered weighted averaging operator

1. INTRODUCTION

Fuzzy inference has been widely used in the field of control system and artificial intelligence. Closed intervals and fuzzy sets have similar effect on representing uncertain data, therefore the Poland school proposed interval analysis[1,2] in the early 1980s. The interval analysis combined with fuzzy set method had better effect, so the concept of interval-valued fuzzy sets (hereinafter referred to as IVFS) was proposed and used in fuzzy inference.

In practical applications, especially in the process of decision, evaluation, *etc.*, due to it's very difficult to grasp the essence of dynamic things, the single value of an object's degree of membership is often not easy to determine, but the interval value of its degree of membership is relatively easier to determine, and interval-valued fuzzy inference method can reduce the loss of information in the inference process. The literature[3] studied two inference forms of simple interval-valued fuzzy inference and multiple interval-valued fuzzy inference on the basis of interval-valued fuzzy relations, but did not consider the condition with uncertainty factors or weights parameters. In the process of inference, due to the influence degree of different factors on the result was not the same, then it accords with the idea of human thinking that the main factors were given bigger weights and secondary factors were given smaller weights. The OWA operator (ordered weighted averaging Operator) theory[4] which was given by the Yager well reflected the above idea.

2. BASIC THEORY ON WEIGHTED FUZZY SETS

In the process of comparing two weighted fuzzy sets, we need some quantitative indexes to denote the compared results. The commonly used indicators are similarity measure and distance, which denote the degree of similarity and difference degree of two weighted fuzzy sets, respectively. This paper gives the calculation formula of similarity measure as follows:

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Definition 1: Assume the $U = \{u_1, u_2, \cdots, u_n\}$ is a universal set, and is a finite set. All weighted fuzzy sets of universal set U are denoted by $IvF(U)$. $A, B \in IvF(U)$ and assume $A = \{(u_i, [A^-(u_i), A^+(u_i)])/u_i \in U\}$, $B = \{(u_i, [B^-(u_i), B^+(u_i)])/u_i \in U\}$, where $A^- : U \to [0,1], A^+ : U \to [0,1]$, and $A^-(u_i) \le A^+(u_i), \forall u_i \in U$. Assume p belongs to the natural number set, *i.e.*, $p \in N^*$. Take $\lambda_i, \mu_i \in [0,1]$ and $\lambda_i + \mu_i = 1$. Assume $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is a weight vector corresponding to the universal set U, where $\omega_i \in [0,1], \sum_{i=1}^n \omega_i = 1, i = 1, 2, \cdots, n$. The similarity measure of A and B is defined as follows:

$$N\left(A(u_i), B(u_i)\right) = 1 - \left\{\sum_{i=1}^{n} \omega_i \left(\lambda_i \left|A^-(u_i) - B^-(u_i)\right|^p + \mu_i \left|A^+(u_i) - B^+(u_i)\right|^p\right)\right\}^{\frac{1}{p}} \tag{1}$$

Specially, for $\mu, \lambda \in [0,1]$ and $\mu + \lambda = 1, p = 2$, the similarity measure of A and B is defined as follows:

$$N\left(A(u_i), B(u_i)\right) = 1 - \left\{\mu \left|A^-(u_i) - B^-(u_i)\right|^2 + \lambda \left|A^-(u_i) - B^-(u_i)\right|^2\right\}^{\frac{1}{2}} \tag{2}$$

About the validity of Definition 1, it must satisfy the four conditions given in literature:[5,6]

(1) $N(A, A) = 1$;

(2) $N(U, \varnothing) = 0, U = (1,1), \varnothing = (0,0)$;

(3) $N(A, B) = N(B, A)$;

(4) $A \subseteq B \subseteq C \Rightarrow N(A,C) \le N(B,C) \wedge N(A,B)$.

The proof of validity of $N(A(U_i), B(U_i))$ is given as follows: $N(A(u_i), B(u_i))$ is given as follows:

Prove: (1)

$$\begin{aligned} N(A, A) &= 1 - \left\{\sum_{i=1}^{n} \omega_i \left(\lambda_i \left|A^-(u_i) - A^-(u_i)\right|^p + \mu_i \left|A^+(u_i) - A^+(u_i)\right|^p\right)\right\}^{\frac{1}{p}} \\ &= 1 - 0 \\ &= 1 \end{aligned}$$

(2) If $A(u_i) = [1,1]$,then $A^C(u_i) = [0,0], i = 1, 2, \cdots, n$, and then there is

$$\begin{aligned} N(A, A^C) &= 1 - \left\{\sum_{i=1}^{n} \omega_i \left(\lambda_i |1-0|^p + \mu_i |1-0|^p\right)\right\}^{\frac{1}{p}} \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

Similarly, if $A(u_i) = [0,0]$, then $A^C(u_i) = [1,1], i = 1, 2, \cdots, n$, and then there is $N(A, A^C) = 0$

(3) $N(A, B) = N(B, A)$ is obvious.

(4)

$\because A \subseteq B \subseteq C$

$\therefore A^-(u_i) \le B^-(u_i) \le C^-(u_i)$, $A^+(u_i) \le B^+(u_i) \le C^+(u_i)$, $i = 1, 2, \cdots, n$,

There is

$\left|A^-(u_i) - C^-(u_i)\right| \ge \left|A^-(u_i) - B^-(u_i)\right|$, $\left|A^+(u_i) - C^+(u_i)\right| \ge \left|A^+(u_i) - B^+(u_i)\right|$

Then

$$\lambda_i \left|A^-(u_i) - C^-(u_i)\right|^p \ge \lambda_i \left|A^-(u_i) - B^-(u_i)\right|^p,$$

$$\mu_i \left|A^+(u_i) - C^+(u_i)\right|^p \ge \mu_i \left|A^+(u_i) - B^+(u_i)\right|^p,$$

$$\begin{aligned} &\sum_{i=1}^{n} \omega_i \left(\lambda_i \left|A^-(u_i) - C^-(u_i)\right|^p + \mu_i \left|A^+(u_i) - C^+(u_i)\right|^p\right) \\ &\ge \sum_{i=1}^{n} \omega_i \left(\lambda_i \left|A^-(u_i) - B^-(u_i)\right|^p + \mu_i \left|A^+(u_i) - B^+(u_i)\right|^p\right) \end{aligned}$$

So,

$$\begin{aligned} N(A, C) &= 1 - \left\{\sum_{i=1}^{n} \omega_i \left(\lambda_i \left|A^-(u_i) - C^-(u_i)\right|^p + \mu_i \left|A^+(u_i) - C^+(u_i)\right|^p\right)\right\}^{\frac{1}{p}} \\ &\le 1 - \left\{\sum_{i=1}^{n} \omega_i \left(\lambda_i \left|A^-(u_i) - B^-(u_i)\right|^p + \mu_i \left|A^+(u_i) - B^+(u_i)\right|^p\right)\right\}^{\frac{1}{p}} \\ &= N(A, B) \end{aligned}$$

Similarly, there is $N(A, C) \le N(B, C)$.

Therefore, the definition of $N(A, B)$ is valid.

Definition 2:[4,7] Assume $F : R^n \to R$, if $F(a_1, a_2, \cdots, a_n) = \sum_{j=1}^n \omega_j b_j$, where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is an n dimensional vector associated with the function $F, \omega_j \in [0,1], j \in \{1, 2, \cdots, n\}, \sum_{j=1}^n \omega_j = 1$, and b_j is the j-th large element in a set of data $(a_1, a_2, \cdots, a_n)$, R is the set of real number, the function F is called n dimensional OWA operator (ordered weighted average operator).

The obvious feature of OWA operator is that firstly reorders the given decision-making data $(a_1, a_2, \cdots, a_n)$ in descending order and obtain a new data $(b_1, b_2, \cdots, b_n)$, and aggregates the new data by the given weight vector. The weight value ω_j has no relationship with the element a_j, it's only related to the j-th position in the aggregate process.

The OWA operator is the aggregation method of multiple attribute decision making information between the maximum and minimum operator. The literature[8] introduced one of the most common methods, that is when $\omega = \left(0, \frac{1}{n-2}, \cdots, \frac{1}{n-2}, 0\right)$. $F(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} \omega_j b_j = \frac{1}{n-2} \sum_{j=2}^{n-1} b_j$. That is, get rid of the maximum and minimum values, and the rest to do the arithmetic average. This method is so concise that it is widely accepted by people, which is usually used for final results of players in tournament. But this method ignored the use of the maximum and minimum values in the decision-making process, and also concealed the respective particularity of decision data. So this paper chooses another kind of interval value weighting method, namely OWA operator weighting method based on combinatorial number. This operator cleverly combined combinatorial number, and had good properties of weight proposed by Xu and Wang.[7,9]

We have given the similarity measure formula of weighted fuzzy set and the weighting method of OWA operators. According to the above definitions, we calculate the comprehensive degree of similarity between the two matched interval value sets according to the following definition 3.

Definition 3: For $A, B \in IvF(U), U = \{u_1, u_2, \cdots, u_n\}$, the calculation formula of comprehensive similarity measure between A and B is

$$S(A,B) = F\left(N\left(A(u_i), B(u_i)\right), N\left(A(u_i), B(u_i)\right), \cdots, N\left(A(u_i), B(u_i)\right)\right)$$

where

$$A(u_i) = \left[A^-(u_i), A^+(u_i)\right], B(u_i) = \left[B^-(u_i), B^+(u_i)\right], i = 1, 2, \cdots, n$$

$N(A(u_i), B(u_i))$ is the similarity measure of interval values $A(u_i)$ and $B(u_i)$.

It is easy to prove that the calculation formula $S(A, B)$ corresponds to the definition of similarity measure and the homologous nature.

3. A WEIGHTED FUZZY INFERENCE METHOD

Consider the weighted fuzzy inference production rules as follows:

The major premises:

R_1: if X is A_1, then Y is B_1, cf_1, λ_1;

R_2: if X is A_2, then Y is B_2, cf_2, λ_2;

$\cdots$

R_n: if X is A_n, then Y is B_n, cf_n, λ_n;

Fact: if X is A^*

Conclusion: then Y is B^*

Where $A_j \in IvF(U), B_j \in IvF(V)$, U and V are universal set of X and Y, respectively, $U = \{u_1, u_2, \cdots, u_n\}, V = \{v_1, v_2, \cdots, v_n\}$. cf_j denotes the credibility of j-th rule, λ_j is the threshold value assigned to the j-th rule, $j = 1, 2, \cdots, n$.

The weighted weighted fuzzy inference based on OWA operator implements the following steps:

Step 1: Domain experts give a set of weighted fuzzy production rules and match facts according to experience, tips and heuristic knowledge.

Step 2: According to the definition 3, we calculate the matching degree vector α_j between the antecedent A_j of j-th rule and the match fact A^*, where $\alpha_{ji} = N(A^*(u_i), A_j(u_i))(j = 1, 2, \cdots, n; i = 1, 2, \cdots, n)$.

Step 3: According to the definition 5, we calculate the weight vector of matching degree vector α_j of j-th rule $\omega_{\alpha_j} = (\omega_{j1}, \omega_{j2}, \cdots, \omega_{jn})$.

Step 4: According to the definition 6, we calculate the comprehensive similarity measure of j-th rule $S_j(A^*, A_j) = F(\alpha_j) = \sum_{i=1}^{n} \omega_{ji}\beta_{ji}$, where β_{ji} is the i-th large data of α_j. Then according to credibility of the rules given by the experts, we calculate the amendatory comprehensive similarity measure $S_j'(A^*, A_j) = s_j(A^*, A_j) \cdot cf_j$ that is correlated with certainty degree.

Step 5: If $S_j'(A^*, A_j) \geq \lambda_j$ then the rule is aroused. Calculate the interval value of inference result of the consequent by the formula $D_j' = \min\left\{[1,1], D_j / S_j'\left(A^*, A_j\right)\right\}$, where D_j is a two-dimension output, *i.e.*, lower limit and upper limit of membership degree. If more than one rule are aroused, calculate the final inference result by the formula $D' = \bigcup_{j=1}^{n} D_j'$. If $S_j'(A^*, A_i) < \lambda_j$, then the rule is not aroused.

4. EXPERIMENT AND ANALYSIS

Example Assume an weighted fuzzy inference system knowledge set includes the following weighted fuzzy production rules:

R_1: if X is A_1, then Y is B_1, $cf_1 = 0.94$, $\lambda_1 = 0.56$;

R_2: if X is A_2, then Y is B_2, $cf_2 = 0.86$, $\lambda_2 = 0.55$;

R_3: if X is A_3, then Y is B_3, $cf_3 = 0.96$, $\lambda_3 = 0.72$;

R_4: if X is A_4, then Y is B_4, $cf_4 = 0.82$, $\lambda_4 = 0.62$.

Where $A_j \in IvF(U)$, $B_j \in IvF(V)$; $U = \{u_1, u_2, u_3, u_4\}$; $V = \{v_1, v_2, v_3\}, cf_j \in [0, 1]$ is a reliability value of j-th rule. $\lambda_j \in [0, 1]$ is a threshold value of j-th rule. Assume the interval value fuzzy set of each rule is:

$A_1=\{(u_1,[0.45,0.63]),(u_2,[0.68,0.89]),(u_3,[0.25,0.48]),(u_4,[0.35,0.50])\}$;

$A_2=\{(u_1,[0.48,0.67]),(u_2,[0.69,0.82]),(u_3,[0.36,0.58]),(u_4,[0.30,0.48])\}$;

$A_3=\{(u_1,[0.76,0.84]),(u_2,[0.52,0.69]),(u_3,[0.61,0.75]),(u_4,[0.66,0.78])\}$;

$A_4=\{(u_1,[0.42,0.70]),(u_2,[0.72,0.96]),(u_3,[0.22,0.48]),(u_4,[0.35,0.54])\}$;

$B_1=\{(v_1,[0.32,0.56]),(v_2,[0.15,0.27]),(v_3,[0.24,0.36])\}$;

$B_2=\{(v_1,[0.44,0.67]),(v_2,[0.75,0.86]),(v_3,[0.36,0.49])\}$;

$B_3=\{(v_1,[0.63,0.85]),(v_2,[0.33,0.56]),(v_3,[0.42,0.61])\}$;

$B_4=\{(v_1,[0.31,0.53]),(v_2,[0.42,0.61]),(v_3,[0.26,0.38])\}$.

The given fact is

$$A^*=\{(u_1,[0.46,0.78]),(u_2,[0.55,0.66]),(u_3,[0.86,0.94]),(u_4,[0.60,0.78])\}$$

According to the aforementioned reasoning steps on Section 5, the inference result of the system can be obtained. The inference is as follows:

Firstly, according to the formula (1), calculate the degree of similarity between the given fact

A^* and the antecedent A_j of the j-th rule is a vector $\alpha_j = (\alpha_{j1},\alpha_{j2},\alpha_{j3},\alpha_{j4}), 1 \le j \le 4$, where $\alpha_{ji} = N(A^*(u_i), A_j(u_i))$. For simplicity, we take $\lambda_i = 2/3, \mu_i = 1/3, 1 \le i \le 4$, there are $\alpha_1 = (0.91, 0.71, 0.44, 0.74), \alpha_2 = (0.93, 0.85, 0.21, 0.70), \alpha_3 = (0.75, 0.97, 0.77, 0.95), \alpha_4 = (0.94, 0.78, 0.41, 0.75)$.

Then, according to the OWA weight method in Ref.[9] and for simplicity, assume the attitude of each expert to every element is the same, the weight of similarity vector α_j of the j-th rule is obtained to be $\omega_j = (0.125, 0.375, 0.375, 0.125), j = 1, 2, 3, 4$. At the same time, we obtain the comprehensive degrees of similarity between the given fact A^* and the antecedent A_j of the j-th rule are $S_1 = 0.71, S_2 = 0.72, S_3 = 0.86, S_4 = 0.74$.

According to the reliability value of each rule given by expert again, calculate the comprehensive correction degrees of similarity associated with degree of certainty

$S_1'=0.67$, $S_2'=0.62$, $S_3'=0.83$, $S_4'=0.61$.

Since $S_1' = 0.67 > \lambda_1 = 0.56$, the rule R_1 is aroused, then the inference result of the rule R_1 is

$$D_1'=\min\left\{[1,1], D_1/0.67\right\}=\{(v_1,[0.48,0.84]),(v_2,[0.22,0.40])(v_3,[0.52,0.81])\}$$

Since $S_2' = 0.62 > \lambda_2 = 0.55$ the rule R_2 is aroused, then the inference result of the rule R_2 is

$$D_2'=\min\left\{[1,1], D_2/0.62\right\}=\{(v_1,[0.71,1]),(v_2,[1,1])(v_3,[0.58,0.79])\}$$

Since $S_3' = 0.83 > \lambda_3 = 0.72$ the rule R_3 is aroused, then the inference result of the rule R_3 is

$$D_3'=\min\left\{[1,1], D_3/0.83\right\}=\{(v_1,[0.76,1]),(v_2,[0.40,0.67])(v_3,[0.51,0.73])\}$$

Since $S_4' = 0.61 > \lambda_4 = 0.62$ the rule R_4 is aroused, then the inference result of the rule R_4 is

$$D'=\bigcup_{j=1}^{3} D_j'=\{(v_1,[0.76,1]),(v_2,[1,1])(v_3,[0.58,0.81])\}$$

This paper studies a fuzzy inference based on fuzzy processing and weighted fuzzy sets , extracts fuzzy geometrical characteristics from the untreated palmprint image by using fuzzy technology for palmprint classification and identification. And when using fuzzy inference to recognise disordered palmprint, these fuzzy geometrical characteristics are the best. The fuzzy geometry feature identification by using the fuzzy inference proposed in this paper is very good.

4.1 Extraction of fuzzy palmprint

This paper extracts fuzzy geometrical characteristics from disordered palmprint image for palmprint classification and identification.

Palmprint selection and data extraction is as follows:

(1) Palmprint selection is generally to select the triangular point place of palmprint. Since this fragment contains a lot of minutiae information, *i.e.* the rotating direction of palmprint curve, length, width, height, breadth, depth, the distance between palmprints and coverage area size. We select overlapping palmprints for target identification. By processing of any two types and direction of five characteristics of palmprint, namely: left rotation, right rotation, bifurcation point, wave vortex, and clear arc on any extent (for example, 20%, 40%, 60%), carry out target recognition.

(2) Extract all the fuzzy geometrical characteristics of palmprint, namely: the length, width, height, breadth, depth, area, ring perimeter, tightness and area coverage metrics.

(3) Using fuzzy geometric feature set as input, train weighted fuzzy sets.

(4) The palmprint in the overlap mode, for a variety of damage (split, blurred, coating ink, coverage and any overlapping), extract simultaneously fuzzy geometrical characteristics from the palmprint image.

(5) Finally, use the destroyed overlap palmprint image to verify the extraction based on weighted fuzzy sets inference and compare the destroyed palmprints with the real.

4.2 Experimental data selection

In here, the main palmprint database used here is the Polytechnic University (PolyU) palmprint Database (The second version), which is constructed in biometric identification research center of Hongkong Polytechnic University. The experimental basic steps are given as follows:

P1. Fifty different palms are chosen from PolyU Palmprint Database, and 3 palmprint images sampled in different time are selected for each palm. These palmprint images are divided into two groups randomly, where the first group is consist of 50 different images that are from 50 palms and is taken as the learning samples database, and the other group is consist of the remaining 100 images and is taken as the test samples database.

P2. The first group images are carried out a preprocessing and feature extraction by the weighted fuzzy inference. The characteristic vectors of palmprint of 50 palms are acquired, which are archived as training samples.

P3. The second group images are used as the test samples to be identified. Firstly, a palmprint image from the second group is chosen randomly, and is carried out the preprocessing and feature extraction with the weighted fuzzy inference. Then the obtained characteristic vectors are matched with those samples in palmprint archive. The random selections and matching recognitions in 300 times are implemented based on the above palmprint recognition.

P4. For each image in the test sample database, after 300 times recognition are done according to the step P3, the times of the correct recognition and error recognition are recorded respectively, and the correct recognition rates are obtained by calculating. For each palmprint image, 10 times repeating experiments are carried out by the step P1 to step P4 based on the weighted fuzzy inference in simulation. The number of samples is different in each experiment.

4.3 Experimental results and discussion

This paper carries out some experiments on the actual operation data. Experimental results show that three characteristics attributes of palmprint, such as height, length and area coverage indicators, are combined as a group as input while being tested, the best results are obtained, as shown in Figure 1.

Using the fuzzy inference method proposed in this paper for image processing, its denoising effect to fuzzy image is better than that of the existing denoising methods. While being tested, to carry out the combination of other fuzzy geometric characteristics, the obtained vector shows that the recognition rate is high. When the input attribute increases, the combination method can improve the correct recognition rate, but sometimes reduce the learning rate of inference, meanwhile also reducing the universality of conclusions.

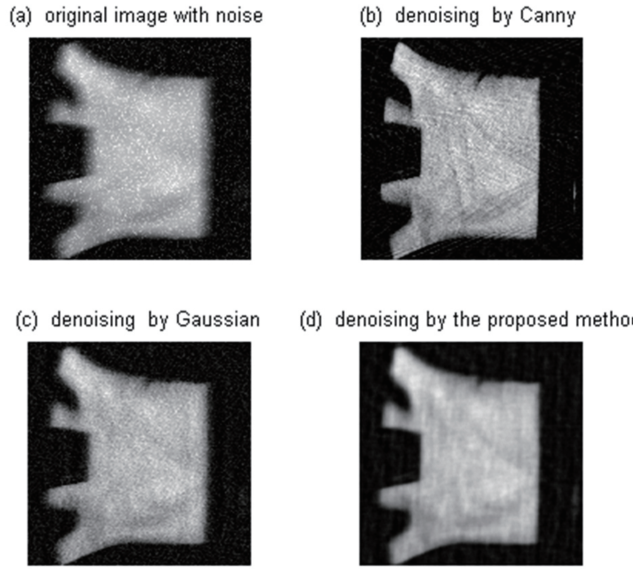

Figure 1. Comparison of image processing method proposed based on fuzzy inference with the existing methods

The experimental results also show that the greater the overlapping degree is, the better the verified image as an overlapping class is. When the palmprint overlap is small, the background of the image mainly needs to be observed. Use different inference, different types of damage for multiple overlapping palmprint images, the experimental results are similar.

From the experiment results, these also show that the most palmprints are able to easily to be recognized in different artificial damages. The degree of recognition is from easy to difficult: the average loss information, dissevered (along or against), besmirched. In the case of coating ink, it produces a worse result.

This paper adopts the fuzzy inference technology of similarity measure of weighted fuzzy sets based on ordered weighted averaging operator for the palmprint data, as shown in Figure 2.

In Figure 2, the abscissa indicates the position of the target operation, and the ordinate indicates the membership size. The solid line indicates the moving trajectory when extracting the characteristics of actual palmprint, and the dotted line indicates the moving trajectory when extracting the characteristics of preprocessing image of the man-made destroyed actual palmprints. Through tracking actually the palmprint data by fuzzy inference based on similarity measure of weighted fuzzy sets, the results show that the inference speed not only has been improved greatly, but also the recognition accuracy of the inference system is also improved greatly.

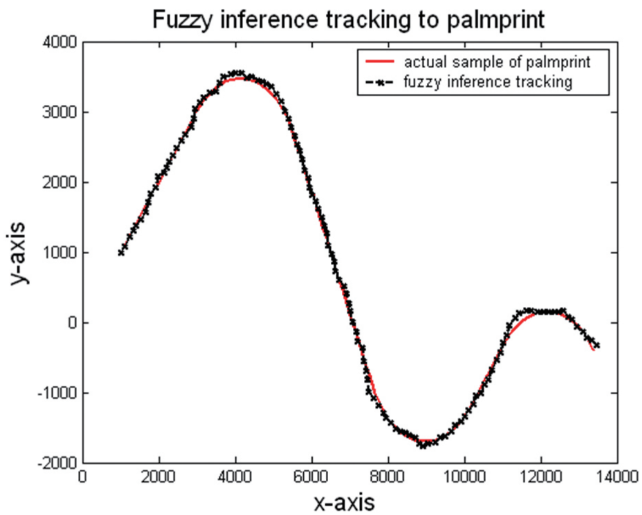

Figure 2. The actually tracking result to palmprint data by fuzzy inference based on similarity measure of weighted fuzzy sets

5. Conclusions

This paper gives an interval valued weighted fuzzy inference method based on OWA operator. We calculate the weight of every weighted fuzzy rule antecedent by a given weight method of OWA operator based on combinatorial number. When calculating the interval valued similarity measure, we obtain a method by distinguishing the importance extent of upper and lower limits of interval value. At the same time, an weighted weighted fuzzy algorithm is proposed. This algorithm takes into account the importance extent of weighted fuzzy inference antecedent to results, and can easily calculate the value of importance extent, so the inference algorithm based on the weighted weight is closer to the actual inference result, and the results are easier to be applied.

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