**ORIGINAL RESEARCH**

# Research on vehicle diagnosis based on state-space method

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## Abstract

In this paper, the vehicle faults are estimated and diagnosed by introducing wavelet transform, based on the state-space method. It monitors the state of railway vehicle suspension system, establishes a vertical dynamic state-space model of railway-vehicle system to identify the parameters of vehicle suspension system. The simulation results show that, in the circumstances of the change of the parameters which is resulted from the gradual fault or the composite fault of the suspension system, the simulation results can effectively identify the basic characteristics of its parameters, realize vehicle fault diagnosis, so as to achieve the purpose of monitoring the state of suspension system, and provide a method for rail vehicle online condition monitoring.

**Key Words:** Vehicle diagnosis, State-space method, Vehicle dynamics, Real-time estimation, MATLAB simulation

## 1 Introduction

Railway vehicle suspension system is the key component of rail vehicles' traveling part, the quality of its state has very important influence on the security and the comfort of vehicles. The traditional diagnoses of rail vehicle state depend on the method of the sensor signal analysis. This method extracts fault features from the signal, mainly based on the statistical relationship between detection signal and fault feature, and analyzes the fault trend by using Expert system and Fuzzy control method. The current commonly used analysis methods include Power Spectral analysis, Correlation analysis of Time-frequency analysis method, and so on. Signal analysis depends more on the statistical analysis of a large number of condition-based monitoring data to determine the fault characteristics and analyze the trend ,and it has some limitations because there are so many sensors are placed on rolling stock.[1,2] Document[3,4] use the Kalman filter to estimate the state value of the vehicle lateral system, in order to get the ideal observations, and the residual value between ideal observational and actual value is used to judge when the system fails. The method can detect the errors in the system, but can do nothing about the obsolescence of the systems. Document[5–7] establishes a model of horizontal state space vehicle system. Under the normal operation state of the vehicle, it uses the algorithm of parameter estimation based on the Particle filter and the Kalman filter to estimate the secondary lateral damping, the secondary Anti-Roll Bar damping and Wheelset equivalent taper. But the article didn't discuss the parameters estimates of the vehicle under fault condition. In this paper, the rail vehicle vertical system state space models are established, and the simulation calculation is performed to identify the variation of the system parameters of rail vehicle suspension. This method provides a solution for the study of rail vehicle fault diagnosis.

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## 2 The establishment of the state equation of vertical vibration of vehicle

### 2.1 The establishment of the vehicle dynamic model

The traditional vertical vibration vehicle system model is shown in Figure 1.

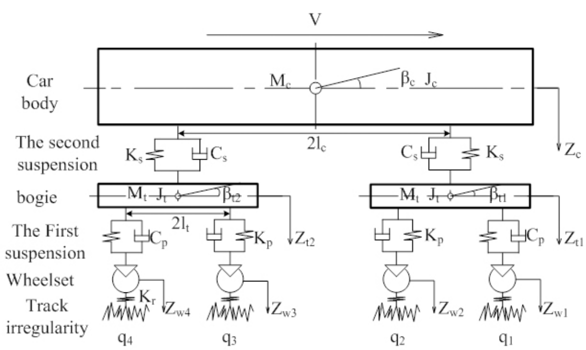



**Figure 1:** The traditional vertical vibration vehicle system model

In Figure 1, assuming that the primary suspension stiffness is the sum of the stiffness of bogie axle box springs, the secondary suspension stiffness is the sum of the stiffness of each spring between the car body and the bogie, damping receives the same treatment.

Vehicle system has 10 degrees of freedom in vertical. Specifically, the body has two degrees of freedom (drifting $z_c$, nod $\beta_c$), each bogie has two degrees of freedom (drifting $z_t$, nod $\beta_t$) and one degree of freedom corresponds to one wheel (ups and downs $z_w$); $M_c$, $M_t$ and $M_w$ are respectively the quality of car body, bogie and wheel pairs; Respectively, the moment of inertia of the body and bogie are shown as nod $J_c$, $J_t$; the primary and secondary vertical damping are shown as $C_p$, $C_s$; $K_p$, $K_s$ respectively correspond to the primary and secondary vertical stiffness. $l_c$ represents the half of the car body distance, $l_t$ represents the half of the framework's wheelbase; $z_c$, $z_{bi}$, $z_{wi}$ are respectively named vertical displacement of car body, bogie and wheel; $\beta_c$, $\beta_t$ are respectively named nod angular displacement of car body and bogie; $q_i$ denotes track's irregularity input.

Differential equation of vertical vibration of vehicle system is shown in the formula (1) $\sim$ (10):

1) Movement of body vibration

$$M_c\ddot{Z}_c + 2C_s\dot{Z}_c + 2K_sZ_c - C_s\dot{Z}_{t1} - C_s\dot{Z}_{t2} - K_sZ_{t1} - K_sZ_{t2} = M_cg \tag{1}$$

2) Movement of body nutation

$$J_c\ddot{\beta}_c + 2C_sl_c^2\dot{\beta}_c + 2K_sl_c2\beta_c + C_sl_c\dot{Z}_{t1} - C_sl_c\dot{Z}_{t2} + K_sl_cZ_{t1} - K_sl_cZ_{t2} = 0 \tag{2}$$

3) The front bogie frame movement vibration

$$M_t\ddot{Z}_{t1} + (C_s + 2C_p)\dot{Z}_{t1} + (K_s + 2K_p)Z_{t1} - C_s\dot{Z}_c - K_sZ_c - C_p\dot{Z}_{w1} - C_p\dot{Z}_{w2} - K_pZ_{w1} - K_pZ_{w2} + C_sl_c\dot{\beta}_c + K_sl_c\beta_c = M_tg \tag{3}$$

4) Movement of the front bogie frame nutation

$$J_t\beta_{t1} + 2C_pl_t^2\dot{\beta}_{t1} + 2K_pl_t^2\beta_{t1} + C_pl_t\dot{Z}_{w1} - C_{pl_t}\dot{Z}_{w1} + K_pl_tZ_{w1} - K_pl_tZ_{w1} = 0 \tag{4}$$

5) Movement of the back bogie frame vibration

$$M_t\ddot{Z}_{t2} + (C_s + 2C_p)\dot{Z}_{t2} + (K_s + 2K_p)Z_{t2} - C_s\dot{Z}_c - K_sZ_c - C_p\dot{Z}_{w3} - C_p\dot{Z}_{w4} - K_pZ_{w3} - K_pZ_{w4} - C_sl_c\dot{\beta}_c - K_sl_c\beta_c = M_tg \tag{5}$$

6) Movement of the back bogie frame nutation

$$J_t\beta_{t2} + 2C_pl_t^2\dot{\beta}_{t2} + 2K_pl_t^2\beta_{t2} + C_pl_t\dot{Z}_{w3} - C_{pl_t}\dot{Z}_{w4} + K_pl_tZ_{w3} - K_pl_tZ_{w4} = 0 \tag{6}$$

7) The first wheelset

$$M_w\ddot{Z}_{w1} + C_p\dot{Z}_{w1} + K_pZ_{w1} - C_p\dot{Z}_{t1} + K_pZ_{t1} + C_pl_t\dot{\beta}_{t1} + K_pl_t\beta_{t1} + P_1(t) - P_0 = 0 \tag{7}$$

8) The second wheelset

$$M_w\ddot{Z}_{w2} + C_p\dot{Z}_{w2} + K_pZ_{w2} - C_p\dot{Z}_{t1} - K_pZ_{t1} - C_pl_t\dot{\beta}_{t1} - K_pl_t\beta_{t1} + P_2(t) - P_0 = 0 \tag{8}$$

9) The third wheelset

$$M_w\ddot{Z}_{w3}+C_p\dot{Z}_{w3}+K_pZ_{w3}-C_p\dot{Z}_{t2}-K_pZ_{t2}+C_pl_t\dot{\beta}_{t2}+K_pl_t\beta_{t2}+P_3(t)-P_0=0 \tag{9}$$

10) The forth wheelset

$$M_w\ddot{Z}_{w4}+C_p\dot{Z}_{w4}+K_pZ_{w4}-C_p\dot{Z}_{t2}-K_pZ_{t2}-C_pl_t\dot{\beta}_{t2}-K_pl_t\beta_{t2}+P_4(t)-P_0=0 \tag{10}$$

Choose the state variables $x$

$$x=[Z_c\ \dot{Z}_c\ \beta_c\ \dot{\beta}_c\ Z_{t1}\ \dot{Z}_{t1}\ \beta_{t1}\ \dot{\beta}_{t1}\ Z_{t2}\ \dot{Z}_{t2}\ \beta_{t2}\ \dot{\beta}_{t2}\ Z_{w1}\ \dot{Z}_{w1}\ Z_{w2}\ \dot{Z}_{w2}\ Z_{w3}\ \dot{Z}_{w3}\ Z_{w4}\ \dot{Z}_{w4}\ ]^T$$

$u(t)$ expresses that vertical track irregular, the formula is

$$u(t)=[Z_{w1}\ \dot{Z}_{w1}\ Z_{w2}\ \dot{Z}_{w2}\ Z_{w3}\ \dot{Z}_{w3}\ Z_{w4}\ \dot{Z}_{w4}]^T$$

### 2.2 Convert the formula of (1) ~ (10) into the form of state space

$$\dot{x}=Ax+f(x(t),u(t)) \tag{11}$$

Among them, $A\in R^{20\times 20}$ is a vehicle system state matrix contains the vehicle mass, damping and stiffness.

Through linearization process of the nonlinear track vibration state,[5] formula (11) can be transformed into the form of formula (12).

$$\dot{x}=Ax+Bu \tag{12}$$

Among them, $B\in R^{20\times 4}$ a matrix of control inputs included the linearized stiffness and the wheel quality. Formula (12) is a continuous function of time $t$. Since the computer is easy to process discrete data, we discretize the formula (12), and the solution to formula (12) is Ref.[9]

$$x(t)=e^{A(t-t_0)}x(t_0)+\int_{t_0}^{t}e^{A(t-\tau)}Bu(\tau)d\tau \tag{13}$$

In the above formula: $t_0$ is the initial time of the sampling interval, assuming that the sampling ends at time $t$; $\tau$ is the time variable of sampling interval between the initial time $t_0$ and $t$. Let's make $T_s$ to be the sampling interval of the $k$th time, the state when $t_0=kT$ is $x(kT)$, then we can use formula (13) to calculate every sampling interval. The state at the time of $t_0$ is

$$x[(k+1)T_s]=e^{AT_s}x(kT_s)+e^{A(k+1)T_s}\int_{kT_s}^{(k+1)T_s}e^{-A\tau}Bu(\tau)d\tau \tag{14}$$

Transform the integral terms of formula (14). During the time interval $[kT_s,(k+1)T_s]$, when $T_s$ is small enough, in other words, the sampling rate is high enough, $u(\tau)$ can be approximately thought as the matrix $u(kT_s)$ consists of constants, then the integral terms become

$$\int_{kT_s}^{(k+1)T_s}e^{-A\tau}Bu(\tau)d\tau=A^{-1}(e^{-AkT_s}-e^{-A(k+1)T_s})Bu(kT_s)=e^{-A(k+1)T_s}A^{-1}(e^{-AT_s}-I)Bu(kT_s) \tag{15}$$

In formula (15), matrix $I$ is a unit matrix. Substitute formula (15) in formula (14), then we can get

$$x[(k+1)T_s]=e^{AT_s}x(kT_s)+A^{-1}(e^{AT_s}-I)Bu(kT_s) \tag{16}$$

Make $A'=e^{AT_s}, B'=A^{-1}(e^{AT_s}-I)B$, formula (16) is converted into

$$x_{k+1}=A'x_k+B'u_k \tag{17}$$

In the above formula: $x_k, x_{k+1}$ respectively represent the state at the time of $kT_s, (k+1)T_s$; $u_k$ represents the value of vertical track irregularity at $kT_s$, we handle it as the Gauss white noise. The vertical accelerations of car body, the front bogie, the rear bogie: $\ddot{z}_c, \ddot{z_{t1}}, \ddot{z_{t2}}$ and the nutation angular acceleration: $\ddot{\beta}_c, \ddot{\beta_{t1}}, \ddot{\beta_{t2}}$ can be obtained through the rail vehicle acceleration sensor.

## 3 Fault diagnosis based on wavelet transform

As for a function with a finite energy $\psi(t)\in L^2(R)$ ($L^2(R)$ represents a square and integrable function space, that is to say a signal space with limited energy), its Fourier transform is $\psi(\tilde{w})$. When $\psi(\tilde{w})$ meets the permissive conditions:

$$C_\psi=\int_{-\infty}^{\infty}\frac{|\psi(\omega)|^2}{|\omega|}d\omega<\infty \tag{18}$$

We call $\psi(t)$ as a basic wavelet or mother wavelet.

The conditional formula (1) implies $\psi(0)$=0, in other words, the functions possess zero mean. We can get a series of wavelet based on the wavelet function $\psi(t)$, after the scale expansion and time displacement, in the form of

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left[\frac{t-b}{a}\right] \tag{19}$$

In the formula, $a$ is a scaling factor, also known as the scale coefficient; $b$ is a time shift factor, also called time position parameters; $a \in R, b \in R, a \neq 0$. For the continuous signal function $f(t) \in L^{(2)}(R)$, the definition of the integral wavelet transform is:

$$W_f(a,b) = \langle f, \psi_{a,b}\rangle = \frac{1}{\sqrt{|a|}}\int_R f(t)\psi\left[\frac{t-b}{a}\right]dt \tag{20}$$

For the discrete signal function $f(t) \in L^{(2)}(R)$, the definition of the integral wavelet transform is:

$$DW_f(\mathrm{t}) = \int_{-\infty}^{\infty} f(t)\psi_{\mathrm{m,\,n}}(t)dt \tag{21}$$

It can greatly reduce the redundancy of wavelet transform and reduce the amount of calculation ($m$ and $n$ are respectively called the frequency range index and time variation of step index).

The wavelet transform has a unique characteristic of time-frequency, and this feature depends on two dimension parameters $a, b$, which changes in field of real number continuously. Among them, time shift factor $b$ affects merely the position of the window on the frequency axis, but $a$ not only affects the position of the window on the frequency axis, but also influences the shape of the window. Thus, wavelet transform can be adjustable for the sampling step in time domain of different frequency. That is to say ,the lower the frequency is, the lower the time resolution of wavelet transform will be, while the higher the frequency resolution of wavelet transform will be; and the same is true the other way around. Thus, it can be seen that, wavelet analysis is suitable for signal processing. Mallat algorithm is a constructor method of orthogonal wavelet basis and a fast algorithm of orthogonal wavelet. According to the Mallat algorithm,[10–12] for any signal $f(t) \in L^{(2)}(R)$, we can decompose the signal $f(t)$ layer by layer. The decomposition result of each layer is making the low frequency signal decomposed into 2 parts: low frequency and high frequency, to make the frequency resolution become higher and higher. As a result, when $a$ is different, the time and frequency resolution are different too, and then, different frequencies of signals can be decomposed by wavelet. The use of multi-scale wavelet transform can extract the frequency characteristics of the signal decomposition.

## 4 Analysis of the results of simulation

Use the above algorithm to recognize the suspension system parameters of rail vehicles during normal running. The relevant parameters are as follows: vehicle parameters are shown in Table 1, The matrix A, B, C can be calculated from these parameters; The initial state $X_0$ is zero; sampling time $T_s = 1$ ms;

**Table 1:** Numerical of railway passenger vehicle parameters

| | | |
|---|---|---|
| Car body quality Mc | kg | 38500 |
| The frame quality Mt | kg | 2980 |
| The quality of wheelset Mw | kg | 1350 |
| Car body nutation inertia Jc | kg·m$^2$ | 2.446*e6 |
| The frame nutation inertia Jt | kg·m$^2$ | 3605 |
| Primary suspension stiffness Ks | N/m | 2.14*e6 |
| Second suspension stiffness Kp | N/m | 2.535*e6 |
| Primary suspension damping Cs | N·s/m | 4.9*e4 |
| Second suspension damping Cp | N·s/m | 1.96*e5 |
| half of Vehicle Length lc | m | 8.4 |
| The half of frame wheelbase lt | m | 1.2 |
| Wheelset radius R | m | 0.4575 |
| Hertz contact coefficient G | \ | (3.86*R$^{-0.115}$)*(10$^{-8}$) |
| The acceleration of gravity g | m/s$^{-2}$ | 9.8 |
| The linearized stiffness Kr | N/m | (1.5/G)*P0$^{1/3}$ |

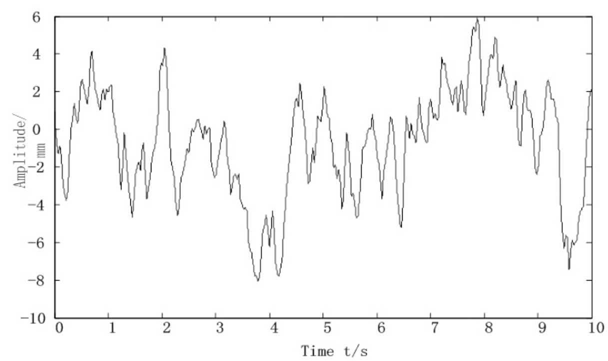
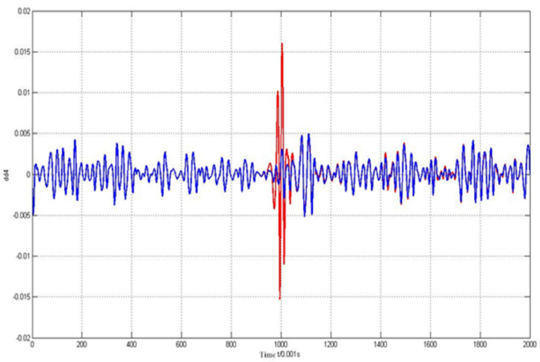



**Figure 2:** Vertical track irregularity input

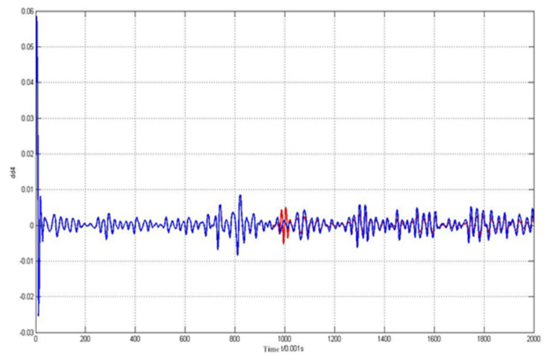
Track static irregularity is one of excitation sources that caused train vibration. In the study of dynamics, track spectrum generally uses the European track irregularity spectrum, the American track irregularity spectrum, Chinese three trunk measured track irregularity spectrum has also been applied in some theoretical study.[13] This paper uses the United States five track irregularity spectrum to simulate functions for track irregularity power spectrum, with the sampling frequency of 10K Hz, and converts it into time domain space track spectrum through Fourier inverse transformation, takes the vertical track irregularity excitation as the model input, just as shown in Figure 2.

### 4.1 Analysis of vehicle gradual fault

Assuming that a vehicle gains outburst failure at time 5 s, primary suspension vertical damping or spring stiffness reduces to its 0.5 times, or second suspension vertical damping or spring stiffness reduces to its 0.5 times. The results are shown in Figures 3-4 (Red waveform is waveform added fault). Use Matlab with db5 wavelet to decompose vertical motion acceleration signal into 6 layers. As can be seen from Figures 3-4, when the signal was decomposed by db5 wavelet, we can clearly see the accurate location of breakpoint from detail signal. It suggests that it is feasible to detect first types of singularity by using wavelet transform.

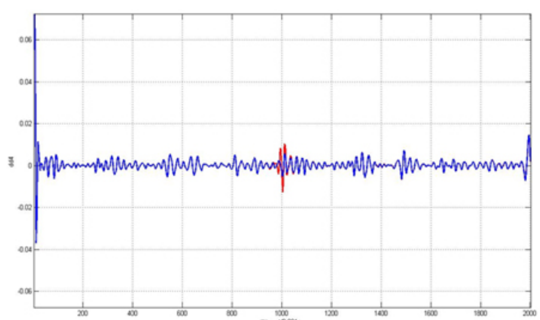


a. Primary suspension vertical stiffness

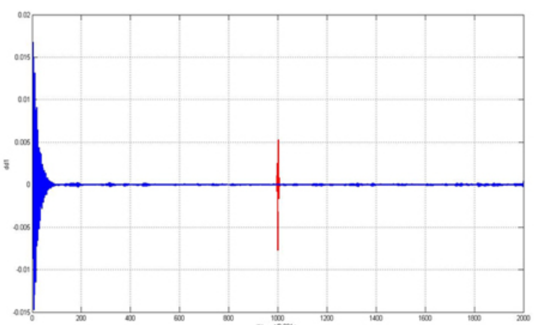


b. Primary suspension vertical damping

**Figure 3:** Vehicle primary suspension gradual fault



a. Second suspension vertical stiffness



b. Second suspension vertical damping

**Figure 4:** Vehicle second suspension gradual fault

As can be seen from Figure 3a, the detail signal on the fourth layer can suggest clearly that the fault occurred in the first second, transient amplitudes of the fault reached $\pm 0.015$. The waveform amplitude after the failure was a bit bigger than the normal waveforms. As can be seen visibly from Figure 3b, the detail signal on the fourth layer can suggest clearly that the fault occurred in the first second, transient amplitudes of the fault reached $\pm 0.015$, the waveform amplitude after the failure was less than the normal waveform.

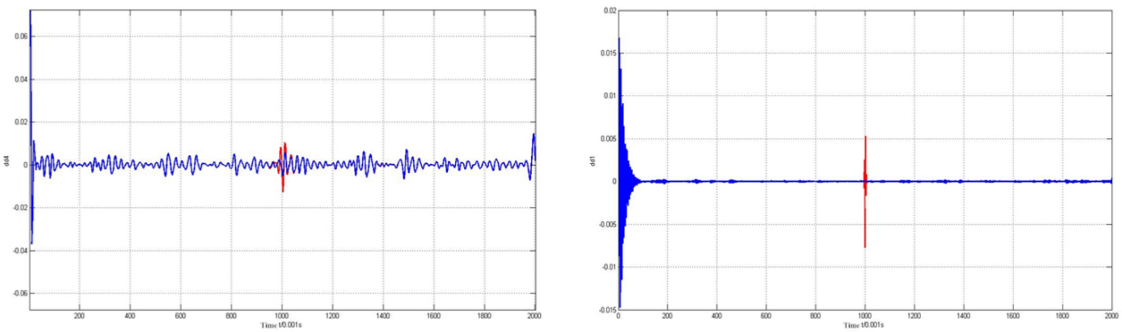
As can be seen from Figure 4a, the detail signal on the fourth layer can suggest clearly that the fault occurred in the first second, transient amplitudes of the fault reached $\pm 0.01$. The wave amplitude and waveform after the failure are in accordance with the normal ones. As can be seen visibly from figure 4b, after the rapid convergence of normal signal, the detail signal on the first layer can suggest clearly that the fault occurred in the first second, transient amplitudes of the fault reached $\pm 0.005$.

Just as clearly shown in Figures 3-4, before the outburst failure, the state of vehicle has reached a steady value, which can show that the vehicle is in normal operation. After the outburst failure, the estimated value of the state changed, this is consistent with the actual running state of the vehicle.

### 4.2 Analysis of vehicle composite fault

In the vehicles' process of service, with the increase of the running time, the phenomenon of aging will happen on some components of the vehicle, causing the gradual change of the dynamics parameters. Assuming that the primary or second suspension vertical damping and spring stiffness of the vehicle reduced to their 0.5 times within a certain period.

Just as clearly shown in Figure 5, the detail signal on the fourth layer can suggest clearly that the fault occurred in the first second, transient amplitudes of the fault reached. The waveform amplitude after the failure was much less than the normal waveform, only reached the half of the normal amplitude. As can be seen visibly from Figure 5b, the detail signal on the fifth layer can suggest clearly that the fault occurred in the first second, transient amplitudes of the fault reached . When the state after failure has become stable, the waveform amplitude became less than the normal wave, about two-thirds of the normal amplitude.



a. Second suspension vertical stiffness    b. Second suspension vertical damping

**Figure 5:** Primary or second suspension vertical stiffness and damping Composite failure

## 5 Conclusions

In this paper, the state monitoring method of the suspension system of railway vehicles is studied, based on wavelet transform algorithm and the vertical state space model of track vehicle system. It discusses the feasibility of the application of wavelet transform algorithm in the field of state-space method for vehicle fault detection and analysis, db5 is chosen as the fundamental point of failure analysis, through the analysis practices of specific waveform, fundamental wavelet is introduced into the analysis of vibration signal under different conditions using the state-space method, as a result, fault can clearly found or judged. It overcomes the fault detection's shortcomings of non-real time. Because of this research is lack of verification of actual test data of real train, the follow-up work should establish a more accurate model of dynamics system to guarantee the accuracy of the vehicle suspension system's state monitoring. At the same time, further research on the results of other fundamental wavelet is necessary.

## Acknowledgements

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