ORIGINAL RESEARCH

Expansion of particle multi-swarm optimization

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ABSTRACT

For improving the search ability and performance of elementary multiple particle swarm optimizers, we, in this paper, propose a series of multiple particle swarm optimizers with information sharing by introducing a special strategy,called multi-swarm information sharing. The key idea, here, is to add a special confidence term into the updating rule of the particle's velocity by the best solution found out by the particle multi-swarm search. This is a new type approach for the technical development and evolution of particle multi-swarm optimization itself. In order to confirm the effectiveness of the information sharing strategy in the proposed particle multi-swarm search, several computer experiments of dealing with a suite of benchmark problems are carried out. For investigating the performance and efficiency of these proposed methods, we compare their search ability and performance, respectively. The obtained experimental results show that the proposed methods have better search ability and performance than those methods without the strategy. And we still decide the best value of adding the new confidence coefficient to the multi-swarm for dealing with the given optimization problems.

Key Words: Swarm intelligence, Particle swarm optimization, Parallel computation, Mixed effect, Information sharing

1. INTRODUCTION

Particle swarm optimization (PSO), created by Kennedy and Eberhart in 1995, is an important technique of stochastic swarm search in the fields of swarm intelligence (SI) and optimization.[1–4] Over the last two decades, a large number of search methods of PSO have been widely developed and applied in different fields of science, economy, engineering, traffic, technology, communication, business, and applications to demonstrate its search ability and performance for dealing with many complex optimization problems and real-world problems.[5–11] The major success factor is that the technique of PSO has three built-in features: (i) information exchange; (ii) intrinsic memory; and (iii) directional search, compared to other existing heuristic and evolutionary techniques such as genetic algorithms (GA), evolutionary programming (EP), evolution strategy (ES), and so on.[12–15] Hence, this stochastic and collective search technique can provide better search results with high-efficiency and high-precision in dealing with various optimization problems.

Generally, the technical development and evolution of PSO can be roughly divided into two trends in formation and methodology. One is the use of a single particle swarm to search, and the other is the use of multiple particle swarms to search. Due to the latter having diversification, expression and efficiency to handle complex optimization problems, these search methods of particle multi-swarm optimization (PMSO, generally, PMSO is just a variant of PSO based on the use of multiple particle swarms [include sub-swarms] instead of a single particle swarm to search.) have been receiving maximal attention among the community of researchers and engineers.[16, 17] It is well-known that many search methods, e.g., a kind of cooperative particle swarm optimizers that are composed of multiple particle swarm op-

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timizers, and hence they still belong to the search methods of PMSO.

In recent years, a lot of studies, reports, and investigations on cooperative particle swarm optimizers that are considered as multiple particle swarms (or sub-swarms) for getting a solution and exchanging certain information during the search process according to some communication strategies, in relation to symbiosis, swarm behavior, and synergy have become popular.[18, 19] Needless to say, when comparing the search ability and performance of particle multi-swarm and particle single swarm, it is natural that the search performance of the former is better than that of the latter besides computation cost, if the configurations of their swarm entities are same, and there are no other constraints. However, from the viewpoint of technical development and evolution, there is room for further research about multi-swarm intelligence and its behavior.

For improving the search ability and performance of elementary multiple particle swarm optimizers, in this paper we propose a series of multiple particle swarm optimizers with information sharing by introducing a special strategy, called multi-swarm information sharing.[20] In the concrete, these proposed methods are multiple particle swarm optimizers with information sharing (MPSOIS), multiple particle swarm optimizers with inertia weight with information sharing (MPSOIWIS), multiple canonical particle swarm optimizers with information sharing (MCPSOIS), and hybrid particle swarm optimizers with information sharing (HPSOIS, HPSOIS has the search characteristics of MPSOIS, MPSOIWIS, and MCPSOIS, which is a firstly proposed search method in the area of PMSO.), respectively.

This is a new type approach for the technical development and evolution of PMSO itself. The key idea, here, is to add the special confidence term into the updating rule of the particle's velocity by the best solution found out by the multi-swarm search. Based on the improvement of confidence term for the multi-swarm search, it is expected to acquire the maximum of potential search ability and performance of the four basic search methods of PMSO under the condition of any adjunctive computation resource.

Specifically, there are three confidence terms in the four basic search methods of PMSO, the detail description in Section 3, to update the particle's velocity. They are the confidence term for the particle, the confidence term for the swarm, and the confidence term for the multi-swarm, respectively. Although the third confidence term is not firstly used in the area of PMSO, but each particle swarm of the multi-swarm is independent to find out the best solution based on the multi-swarm information sharing for the first time. Regarding this

point, there is the novelty, i.e. nothing likes the relation of master-slave model,[21] in the multi-swarm construction and information transmission.

To investigate the search ability and performance of the proposed methods, we try to examine the search effect of every search method with the strategy of multi-swarm information sharing. Under this situation, the selection of positive or zero or negative confidence terms for the multi-swarm to change their search behaviors is possible. As a remarkable result, the given value of the new confidence coefficient can bring a qualitative change in the confidence relationship. Moreover, it is predicted that the strategy will greatly affect the search process and final results for dealing with the given optimization problems.

The rest of this paper is organized as follows: Section 2 briefly introduces the three basic search methods of PSO. Section 3 minutely describes the four basic search methods and their characteristics of PMSO. Section 4 implements several computer experiments and result analysis for investigating the search ability and performance of these proposed methods, and decides the best value of adding the new confidence coefficient. Finally, the concluding remarks appear in Section 5.

2. BASIC SEARCH METHODS OF PSO

Despite the fact that there are many search methods derived from the technique of PSO, e.g., Combinatorial PSO,[22] Fuzzy PSO,[23] Genetic PSO,[24] Niche PSO[25] etc., they have all evolved and developed from the following three basic search methods of PSO.[26]

For the sake of convenience to description for the three basic search methods of PSO, which are the particle swarm optimizer (the PSO),[1, 2] particle swarm optimizer with inertia weight (PSOIW),[27, 28] and canonical particle swarm optimizer (CPSO).[29, 30] Without losing generality, in this paper, let the search space be N-dimensional, $\Omega \in \Re^N$, the number of particles in each swarm be Y, the position (i.e. solution) of the i-th particle be $\vec{x}^i = (x_1^i, x_2^i, \cdots, x_N^i)^T$, and its velocity (i.e. variation) be $\vec{v}^i = (v_1^i, v_2^i, \cdots, v_N^i)^T$, respectively.

2.1 Mechanism of the PSO

The original particle swarm optimizer is proposed by Kennedy & Eberhart. Here, the stochastic and collective search method is referred to as the PSO. For beginning of the PSO search, the position and velocity of the i-th particle are generated at random, then the states of each particle is updated as follows:

$$\vec{x}_{k+1}^i = \vec{x}_k^i + \vec{v}_{k+1}^i \qquad (1)$$

$$\vec{v}_{k+1}^{i} = w_0 \vec{v}_k^{i} + w_1 \vec{r}_1 \otimes (\vec{p}_k^{i} - \vec{x}_k^{i}) + w_2 \vec{r}_2 \otimes (\vec{q}_k - \vec{x}_k^{i}) \quad (2)$$

where w_0 is an inertia weight, w_1 is a confidence coefficient for individual confidence, w_2 is a confidence coefficient for swarm confidence. $\vec{r}_1, \vec{r}_2 \in \Re^N$ are two random vectors in which each element is uniformly distributed over the range $[0, 1]$, and the symbol $\otimes$ is an element-wise operator for vector multiplication. $\vec{p}_k^{i}$ is the local best position of the i-th particle up to now, and $\vec{q}_k$ is the global best position among the whole particle swarm.

In the original PSO, the inertia weight $w_0 = 1.0$ and the confidence coefficients $w_1 = w_2 = 2.0$ are used.[2] Since $w_0 = 1.0$ is set, the convergence of the PSO is not so good in its search process. The PSO has the characteristics of global search.

2.2 Mechanism of PSOIW

Due to overcome the convergence and improve search ability and performance of the PSO, Shi & Eberhart modified the updating rule of the particle's velocity in Eq.(2) by constant reduction of the inertia weight over time-step as follows:

$$\vec{v}_{k+1}^{i} = w(k) \vec{v}_k^{i} + w_1 \vec{r}_1 \otimes (\vec{p}_k^{i} - \vec{x}_k^{i}) + w_2 \vec{r}_2 \otimes (\vec{q}_k - \vec{x}_k^{i}) \quad (3)$$

where $w(k)$ is a variable inertia weight that is linearly reduced from a starting value, w_s, to a terminal value, w_e, with the increment of time-step k given by

$$w(k) = w_s + \frac{w_e - w_s}{K} k \quad (4)$$

where K is the maximum number of time-step for PSOIW search. In the original PSOIW, the boundary values are adopted to $w_s = 0.9$ and $w_e = 0.4$, respectively, and confidence coefficients $w_1 = w_2 = 2.0$ are still used as in the PSO. Since the linearly change of inertia weight from 0.9 to 0.4, PSOIW has the characteristics of asymptotical/local search.

2.3 Mechanism of CPSO

For same purpose as the previous described in Section 2.2, Clerc & Kennedy modified the updating rule as well for the particle's velocity in Eq.(2) by a constant inertia weight over time-step as follows:

$$\vec{v}_{k+1}^{i} = \Phi \left(\vec{v}_k^{i} + w_1 \vec{r}_1 \otimes (\vec{p}_k^{i} - \vec{x}_k^{i}) + w_2 \vec{r}_2 \otimes (\vec{q}_k - \vec{x}_k^{i}) \right) \quad (5)$$

where Φ is an inertia weight corresponding to w_0. In the original CPSO, $\Phi = w_0 = 0.729$, confidence coefficients $w_1 = w_2 = 2.05$ are used.

It is clear that since the value of inertia weight of CPSO is smaller than 1.0, the convergence of its search is guaranteed by compared with the PSO search.[31, 32] CPSO has the characteristics of local search.

3. BASIC SEARCH METHODS AND CHARACTERISTICS OF PMSO

Formally, there are a lot of search methods about PMSO.[17] For understanding the formation and methodology of these proposed methods, let us assume that the multi-swarm consists of multiple single swarms. The corresponding three kinds of particle swarm optimizers described in Section 2 can be generated by construction and parallel computation.[33] Therefore, these constructed particle multi-swarm optimizers, i.e. multiple particle swarm optimizers (MPSO), multiple particle swarm optimizers with inertia weight (MPSOIW), multiple canonical particle swarm optimizers (MCPSO), and hybrid particle swarm optimizers (HPSO, HPSO is a mixed search method which has the search characteristics of the PSO, PSOIW, and CPSO.) are easily acquired by programming. However, there are two confidence terms in the Eqs.(2), (3) and (5) to be used in their updating rule.

Generally, they are called as the elementary basic search methods of PMSO which have the updating rule of the particle's velocity.[34, 35]

3.1 Basic search methods of PMSO

For improving the search ability and performance of the above described elementary multiple particle swarm optimizers, furthermore, we add the special confidence term into the updating rule of the particle's velocity by the best solution found out by the multi-swarm search, respectively. This is a part of preparation for PMSO deployment on the next foundation. According to this extended procedure, as the four basic search methods of PMSO, i.e. MPSOIS, MPSOIWIS, MCPSOIS, and HPSOIS can be constructed.[36] Consequently, these basic search methods of PMSO augmented with the strategy of multi-swarm information sharing are proposed.

3.1.1 Mechanism of MPSOIS

On basis of the above description of PMSO, as the mechanism of the proposed MPSOIS, the updating rule of each particle's velocity of the method is defined as follows:

$$\begin{aligned} \vec{v}_{k+1}^{i} = {}& w_0 \vec{v}_k^{i} + w_1 \vec{r}_1 \otimes (\vec{p}_k^{i} - \vec{x}_k^{i}) + w_2 \vec{r}_2 \otimes (\vec{q}_k - \vec{x}_k^{i}) \\ & + w_3 \vec{r}_3 \otimes (\vec{s}_k - \vec{x}_k^{i}) \end{aligned} \quad (6)$$

where $\vec{s}_k$ is the best solution chosen from the best solution set of the whole particle swarms, w_3 is a new confidence coefficient for the multi-swarm, and $\vec{r}_3$ is a random vector in which each element is uniformly distributed over the range $[0, 1]$.

3.1.2 *Mechanism of MPSOIWIS*

In same way as the mechanism of MPSOIS, the updating rule of each particle's velocity of the proposed MPSOIWIS is defined as follows:

$$\vec{v}_{k+1}^i = z(k)\,\vec{v}_k^i + w_1\vec{r}_1 \otimes (\vec{p}_k^i - \vec{x}_k^i) + w_2\vec{r}_2 \otimes (\vec{q}_k - \vec{x}_k^i)$$
$$+ w_3\vec{r}_3 \otimes (\vec{s}_k - \vec{x}_k^i) \qquad (7)$$

Since Eq.(3) and Eq.(6) are alike in formulation, the description of the symbols in Eq.(7) is omitted.

3.1.3 *Mechanism of MCPSOIS*

Similar to the mechanism of MPSOIS, the updating rule of each particle's velocity of the proposed MCPSOIS is defined as follows:

$$\vec{v}_{k+1}^i = \Phi\Big(\,\vec{v}_k^i + w_1\vec{r}_1 \otimes (\vec{p}_k^i - \vec{x}_k^i) + w_2\vec{r}_2 \otimes (\vec{q}_k - \vec{x}_k^i)$$
$$+ w_3\vec{r}_3 \otimes (\vec{s}_k - \vec{x}_k^i)\Big) \qquad (8)$$

Likewise, the description of the symbols in Eq.(8) is omitted.

3.1.4 *Mechanism of HPSOIS*

Based on the three basic particle swarm optimizers that are described in Section 2, there are the three updating rule of each particle's velocity in the proposed HPSOIS. They are given as Eqs.(6), (7) and (8).

Due to the mixed effect and performance in whole search process, global search and asymptotical/local search are implemented simultaneously for dealing with a given optimization problem. It is obvious that HPSOIS has search characteristics of the above-stated basic search methods, i.e. MPSOIS, MPSOIWIS, and MCPSOIS.

3.2 Characteristics of PMSO

For indicating the image relation of the above described search methods, Figure 1 simply shows the constitutional concept of the proposed four basic search methods of PMSO. It is clear that MPSOIS, MPSOIWIS, and MCPSOIS are parallel computation with same search method, respectively. However, HPSOIS is a mixed search method which is composed of PSOIS, PSOIWIS and CPSOIS. So it has different characteristics of the above search methods as a new basic search method of PMSO.[36]
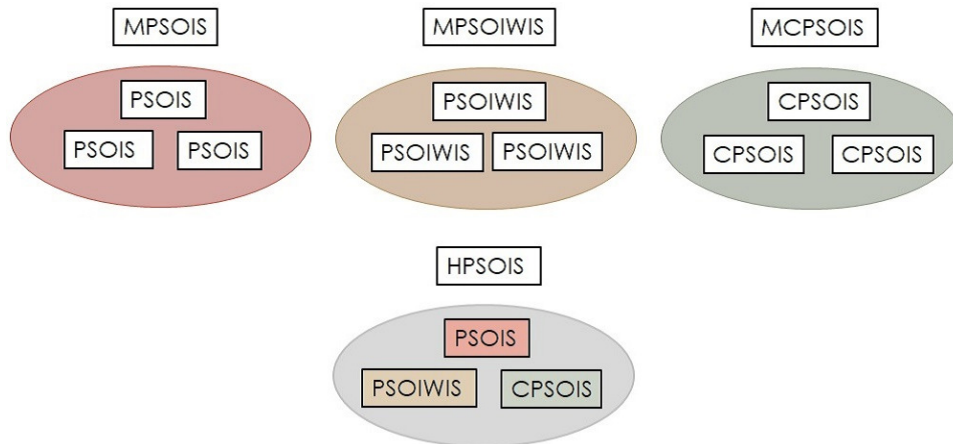

Figure 1. The constitutional concept of the proposed four basic search methods of PMSO

Specifically, under the situation of multi-swarm search, the best solutions are obtained from every particle swarm. Then, the most best solution can be determined from the set of these best solutions. This most best solution is what we obtain from the particle multi-swarm search. According to the updating rule of conventional particle's velocity of the multi-swarm, we add the special confidence term for the multi-swarm into the updating rules of the particle's velocity to reinforce the search ability and performance. It is clear that the added term perfectly is in accordance with the fundamental construction principle of PSO.

Figure 2 shows the search characteristics of the proposed four basic search methods of PMSO with the variation of the confidence coefficient w_3 for a particle's flying. We can see that if $w_3 > 0$, third confidence term plays a cooperative role to update the particle's velocity. In contrast with the former, if $w_3 < 0$, the confidence term plays an inhibitive role to update the particle's velocity.

Based on the selected value (i.e. positive or zero or negative) of the confidence coefficient w_3, totally different search features and effects will occur. Especially, when $w_3 = 0$, since the third confidence term in these basic search methods becomes zero, it is same as the case, i.e. the basic search methods without information sharing. Therefore, these search methods corresponding to the most basic search ones which are MPSO, MPSOIW, MCPSO, and HPSO, respectively.
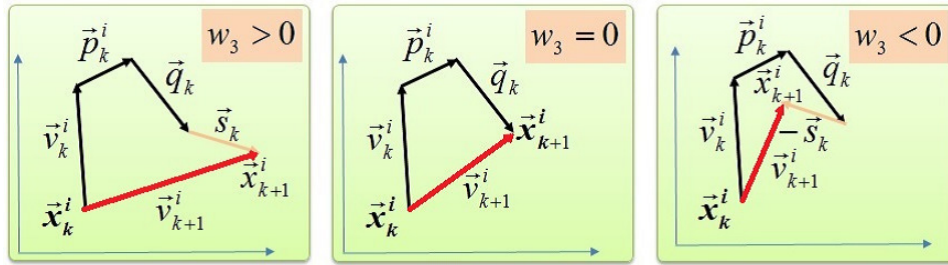
Figure 2. The variation of the confidence coefficient w_3 for a particle's flying

Regarding the convergence of the above search methods, it can be said that the MPSOIS has the characteristics of global search, MPSOIWIS has the characteristics of asymptotical/local search, and MCPSOIS has the characteristics of local search. With different search features, focusing on the mixed effect of the three basic particle swarms, and create HPSOIS that has higher search ability and diversity, so it is expected to improve the potential search ability and performance of PMSO to be maximum without additional calculation resouce.

4. COMPUTER EXPERIMENTS AND RESULT ANALYSIS

To investigate the search ability and performance of the proposed methods, i.e. MPSOIS, MPSOIWIS, MCPSOIS, and HPSOIS, several computer experiments are carried out for dealing with a suite of benchmark problems. Table 1 shows the selected four functions (include two unimodals and two multimodals) of the given optimization problems.[37]

Table 1. A suite of benchmark problems for computer experiments

problem	function
Sphere	$f_{Sp}(\vec{x}) = \sum_{d=1}^{N} x_d^2$
Schwefel	$f_{Sw}(\vec{x}) = \sum_{d=1}^{N} \left(\sum_{j=1}^{d} x_j \right)^2$
Griewank	$f_{Gr}(\vec{x}) = \frac{1}{4000} \sum_{d=1}^{N} x_d^2 - \prod_{d=1}^{N} \cos\left(\frac{x_d}{\sqrt{d}}\right) + 1$
Rastrigin	$f_{Ra}(\vec{x}) = \sum_{d=1}^{N} \left(x_d^2 - 10\cos\left(2\pi x_d\right) + 10 \right)$

To obtain the biggest value of the given functions for handling these optimization problems, the criterion of the solution expressed by the following equation is used:

$$g_*(\vec{x}) = \frac{1}{1 + f_*(\vec{x})} \qquad (9)$$

where the symbol $*$ refers to Sp, Sc, Gr and Ra of the respective problems.

By dealing with these given optimization problems, we minutely investigate the basic characteristics and search performance of these proposed methods. Table 2 gives the selected values of the main parameters used in performing the four basic search methods of PMSO.

Table 2. Main parameters for performing the proposed search methods of PMSO

parameter	value
number of the used swarms, S	3
number of particles in a swarm, Y	10
number of particle swarm search, K	400
number of dimension for searching, D	6, 10
confidence coefficient to multi-swarm, w_3	-1.0~1.0

Computing environment and the used software in our experiments are as follows:

DELL: OPTIPLEX 3020, Intel(R) core(TM) i5-4590, CPU 3.30GHz, RAM 8.0GB.

Mathematica: version 11.3.

4.1 Search process

For easy to observe the variation of search process of every proposed method, the value of the confidence coefficient w_3 is set at 1.0 or 0 or -1.0, respectively, we implement several computer experiments to deal with the $Rastragin$ problem in 6D case. The investigation and analysis on the change in the characterisitics of the four proposed methods are carried out as follows.

4.1.1 Search process of MPSOIS

Figure 3 shows the obtained different search processes by performing MPSOIS with adjusting the value of the confidence coefficient w_3 (Computing time is about 3.4 sec. to each search for dealing with the given optimization problem.). The Best1 and Mean1 shown in Figure 3 refer to the best solution and average solution obtained by the No.1 particle swarm. As the same way, Best2 and Mean2, Best3

and Mean3 refer to the best solutions and average solutions obtained by the No.2 and No.3 particle swarms, respectively.

It can be seen that when $w_3 = 1.0$ is set, all of the three particle swarms rapidly found out the best solution (i.e. fitness value is 1.0 at the position). This effect is evidence that plays a cooperative role. When $w_3 = 0$ is set, regardless of the best solution is found out by three particle swarms, their convergence to the best solution is not so fast than that in $w_3 = 1.0$ case. This result shows the difference between MP-SOIS and MPSO in search ability and performance. When $w_3 = -1.0$ is set, the three particle swarms yield different solutions, and two particle swarms do not arrive at the best solution contrary to the search results in Figure 3(a) and (b). It is clear that the characteristics of search process, shown in Figure 3(a), (b) and (c), are quite different with regard to the variation of the confidence coefficient w_3. Moreover, it is suggested that the negative value is used in the information sharing strategy.
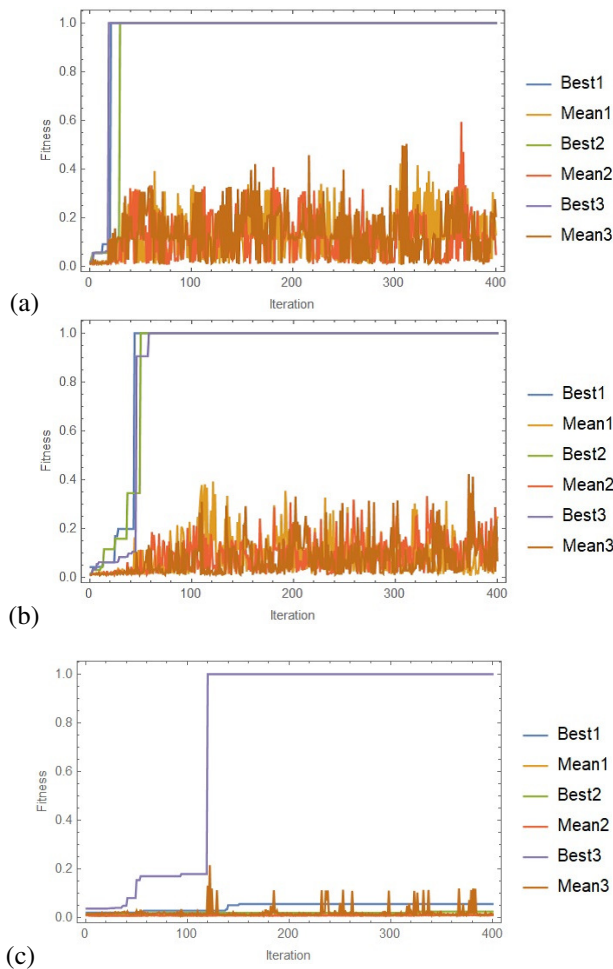
The variation of average solutions in Figure 3 shows that MPSOIS has the characteristics of global search, which has strong robustiousness to treat the variation of the confidence coefficient w_3 in the whole search process.

4.1.2 *Search process of MPSOIWIS*

Figure 4 shows the obtained different search processes by performing MPSOIWIS with adjusting the value of the confidence coefficient w_3.
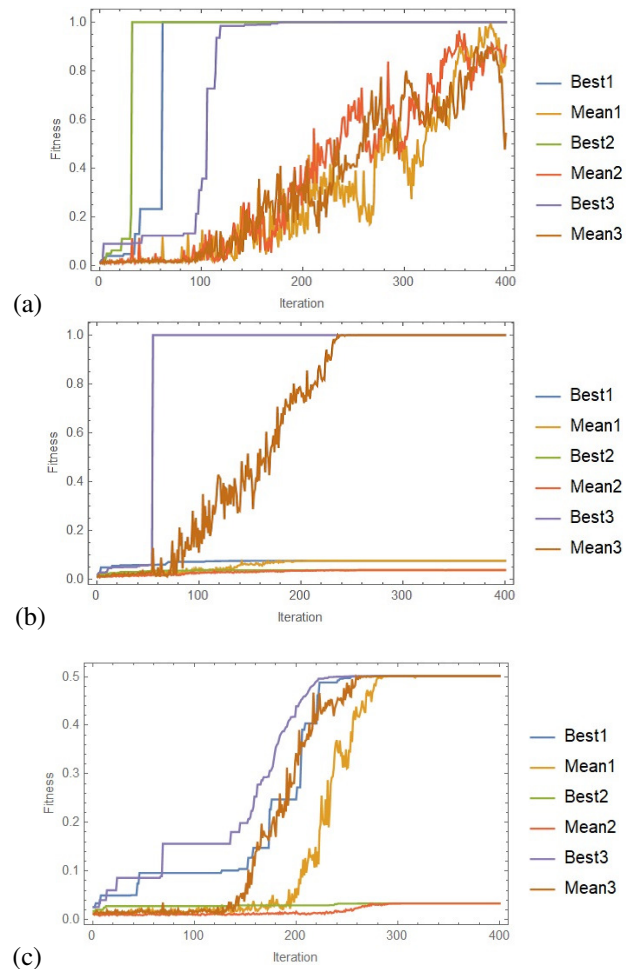

(a)

(b)

(c)

Figure 4. The search process for handling the $Rastrigin$ problem in D6 by performing MPSOIWIS. (a) $w_3 = 1.0$ case, (b) $w_3 = 0$ case, (c) $w_3 = -1.0$ case.

(a)

(b)

(c)

Figure 3. The search process for handling the $Rastrigin$ problem in D6 by performing MPSOIS. (a) $w_3 = 1.0$ case, (b) $w_3 = 0$ case, (c) $w_3 = -1.0$ case.

It can be seen that all of the three particle swarms found out the best solution (i.e. fitness value is 1.0 at the position) when $w_3 = 1.0$ is set. This effect is evidence that plays a cooperative role. When $w_3 = 0$ is set, the best solution is found out by only one particle swarm, and its convergence to the best solution is not so fast than that in $w_3 = 1.0$ case. This result shows the difference between MPSOIWIS and MPSOIW in search ability and performance. When $w_3 = -1.0$ is set, the three particle swarms yield different solutions, and these particle swarms do not arrive at the best solution contrary

to the search results in Figure 4(a) and (b). This effect is evidence that plays an inhibitive role.

It is clear that the search ability and performance of MP-SOIWIS is lower than that of MPSOIS by comparison. The variation of average solutions in Figure 4 shows that MPSOI-WIS has the characteristics of asymptotical/local search in the whole search process.

The obtained results represent that global search is better than asymptotical/local search under the situation of multi-swarm search.

4.1.3 *Search process of MCPSOIS*

Figure 5 shows the obtained different search processes by performing MCPSOIS with adjusting the value of the confidence coefficient w_3.
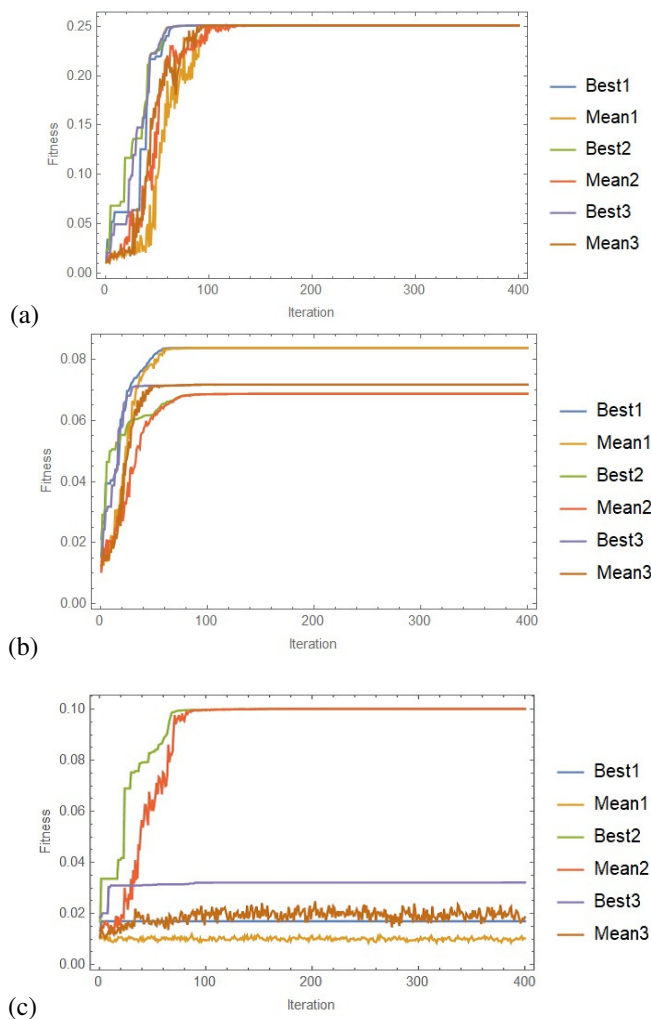
(a)

(b)

(c)

Figure 5. The search process for handling the $Rastrigin$ problem in D6 by performing MCPSOIS. (a) $w_3 = 1.0$ case, (b) $w_3 = 0$ case, (c) $w_3 = -1.0$ case.

It can be seen that all of the three particle swarms found out different solutions (i.e. their fitness values are not 1.0 at the positions) when $w_3 = 1.0$ is set. When $w_3 = 0$ is set, different solutions are found out by the particle swarms, their fitness values are lower than that in $w_3 = 1.0$ case. This result shows the difference between MCPSOIS and MCPSO in search ability and performance. When $w_3 = -1.0$ is set, the three particle swarms yield different solutions, and these particle swarms do not arrive at the best solution contrary to the search results in Figure 3(a).

It is clear that since the convergence of MCPSOIS is so stronger, so the search method leads particle swarms search and makes them to fall into local minimum. Generally, the search ability and performance of MCPSOIS is lower than that of MPSOIS and MPSOIWIS.

The variation of average solutions in Figure 5 shows that MCPSOIS has the characteristics of local search in the whole search process.

4.1.4 *Search process of HPSOIS*

Figure 6 shows the obtained different search processes by performing HPSOIS with adjusting the value of the confidence coefficient w_3.

It can be seen that all of the three particle swarms found out different solutions (i.e. their fitness values are not 1.0 at the positions) when $w_3 = 1.0$ is set. When $w_3 = 0$ is set, different solutions are found out by the particle swarms, their fitness values are lower than that in $w_3 = 1.0$ case. This result shows the difference between MCPSOIS and MCPSO in search ability and performance. When $w_3 = -1.0$ is set, the three particle swarms yield different solutions, and these particle swarms do not arrive at the best solution contrary to the search results in Figure 3(a).

It is clear that since the convergence of MCPSOIS is so stronger, so the search method leads particle swarms search and makes them to fall into local minimum. Generally, the search ability and performance of MCPSOIS is lower than that of MPSOIS and MPSOIWIS.

The variation of average solutions in Figure 5 shows that MCPSOIS has the characteristics of local search in the whole search process.

4.1.5 *Search process of HPSOIS*

Figure 6 shows the obtained different search processes by performing HPSOIS with adjusting the value of the confidence coefficient w_3.
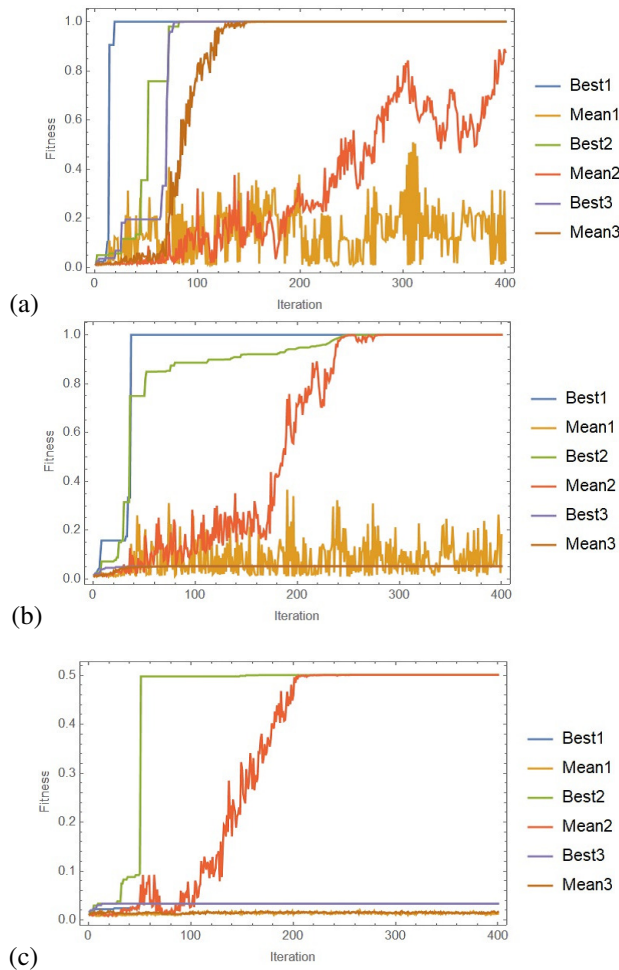
Figure 6. The search process for handling the $Rastrigin$ problem in D6 by performing HPSOIS. (a) $w_3 = 1.0$ case, (b) $w_3 = 0$ case, (c) $w_3 = -1.0$ case.

From Figure 6(a), it can be seen that all of the three particle swarms found out the best solution (i.e. fitness value is 1.0 at the position) when $w_3 = 1.0$ is set. This effect is evidence that plays a cooperative role. When $w_3 = 0$ is set, the best solution is found out by two particle swarms, their fitness values arrive at 1.0. However, their behaviors are later than that in $w_3 = 1.0$ case. This result shows the difference between HPSOIS and HPSO in search ability and performance. When $w_3 = -1.0$ is set, the three particle swarms yield different solutions, and these particle swarms do not arrive at the best solution contrary to the search results in Figure 6(a) or (b). This effect is evidence that plays an inhibitive role.

The variation of average solutions in Figure 6 shows that HPSOIS has the characteristics of global search and asymptotical/local search in the whole search process.

As the above observed experimental results, Figures 3~6 clearly show that how about the search features and effect of cooperative search behaviors of MPSOIS, MPSOIWIS,

MCPSOIS, and HPSOIS based on the special strategy of multi-swarm information sharing. Therefore, the exploratory behavior is controlled by the selected value of the confidence coefficient w_3. So the search ability and performance of the proposed methods with adjusting the value of the confidence coefficient w_3 is further investigated quantitatively.

4.2 Search performance

We examine the search ability and performance of the proposed methods by implementing a suite of benchmark problems in 6D and 10D cases for comparison and analysis. In each computer experiment, the selected value of the confidence coefficient w_3 is changed from -1.0 to 1.0 for the observation on search results.

Under the same computational conditions given in Table 2, the experimental results (i.e. the average of the best solutions obtained by performing every search method 10 times) are obtained for dealing with the given benchmark problems in 6D and D10 cases with performing MPSOIS, MPSOIWIS, and HPSOIS, respectively. Because MCPSOIS is decline in search ability and performance shown in Figure 5, here, we do not continue to check up its experimental results in the following.

Observing these experimental results shown in Figure 7, the search ability and performance of performing MPSOIS (M1), MPSOIWIS (M2), and HPSOIS (Mx) can be summarized as follows:

(1) For handling the $Sphere$ problem, it turns out that all of the proposed methods have succeeded in obtaining the best solution, irrespective of adjusting the value of the confidence coefficient w_3.

(2) For handling the $Schwefel$ problem, it turns out that the result of each proposed method against the obtained the best solution almost irrespective of adjusting the value of the confidence coefficient w_3.

(3) For handling the $Griewank$ problem, it turns out that the performance of each proposed method nearly increases with raising the value of the confidence coefficient w_3. Towards the end of search process, it shows a slightly decreasing tendency.

(4) For handling the $Rastrigin$ problem, it is found that the fitness value gradually increases with adjusting the value of the confidence coefficient w_3. When w_3 exceeds a certain value, the fitness value of performing MPSOIWIS starts to decrease except for the MPSOIS and HPSOIS.

Due to the above results of elementary investigation, the search of MPSOIWIS is hardly affected by the variation of the confidence coefficient w_3. The search ability and performance for handling the multimodal problems are found out to be a non-monotone increasing unlike the search feature of handling unimodal problems.
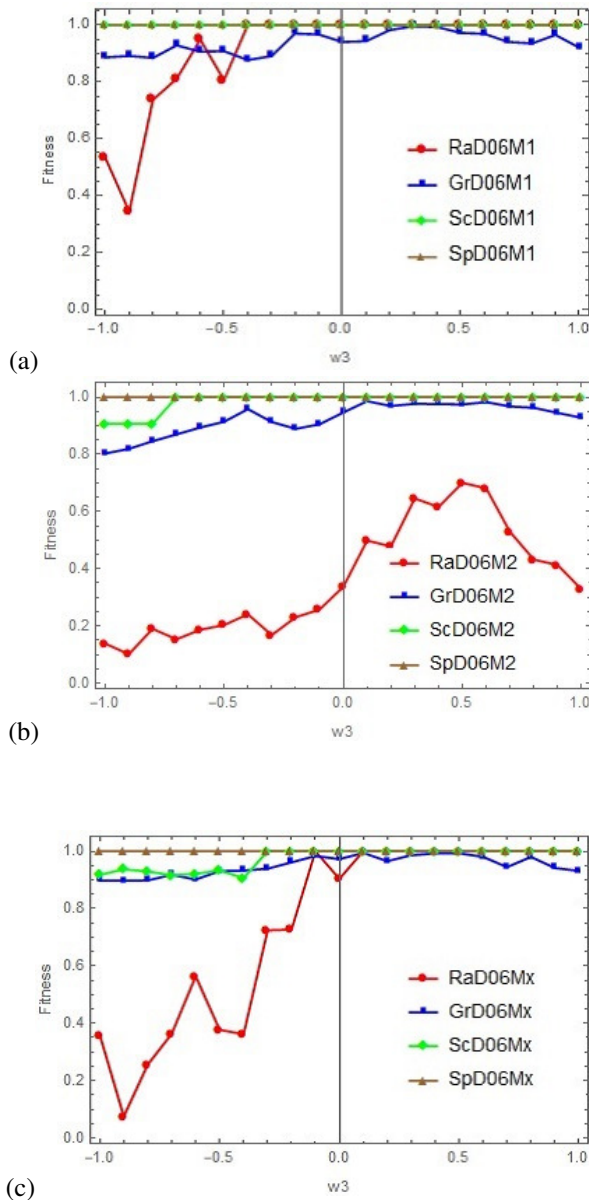

(a)

(b)

(c)

Figure 7. The search results on the average 10 times for dealing with the proposed methods in 6D case. (a) MPSOIS (M1), (b) MPSOIWIS (M2), (c) HPSOIS (Mx).

Based on verifying these obtained experimental results, under the same implementing conditions, several experiments are carried out for dealing with the given benchmark problems in 10D case to confirmation. The obtained experimental results are shown in Figure 8.
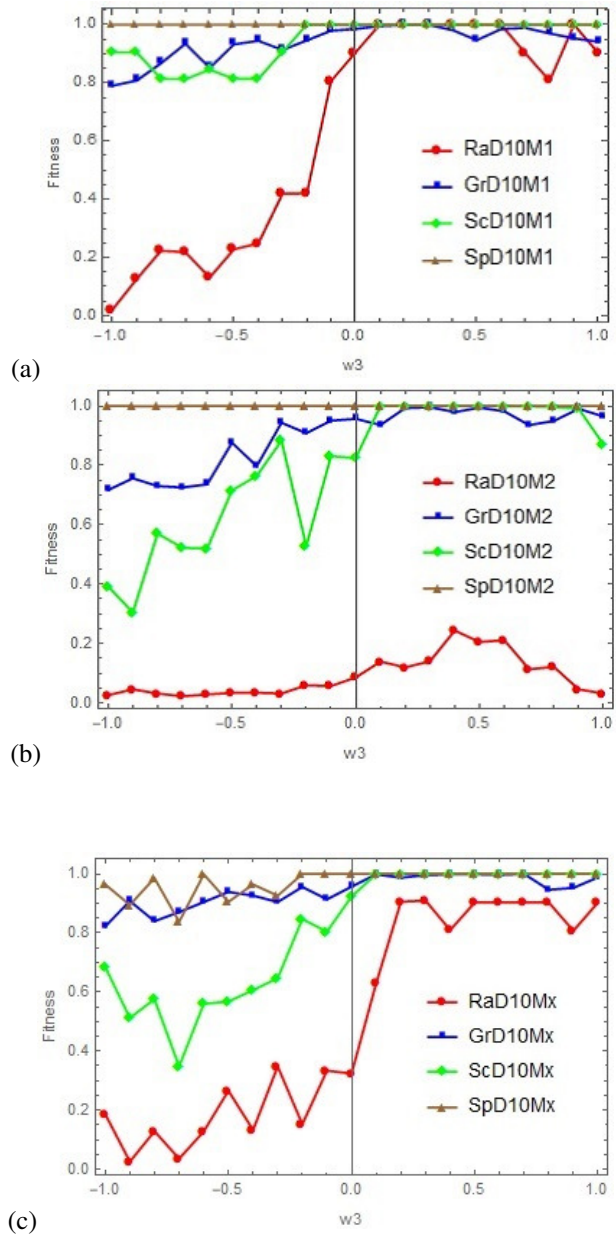

(a)

(b)

(c)

Figure 8. The search results on the average 10 times for dealing with the proposed methods in 10D case. (a) MPSOIS (M1), (b) MPSOIWIS (M2), (c) HPSOIS (Mx).

By comparing the search ability and performance of the three methods that are shown in Figures 7 and 8, the following findings can be summed up as follows.

First, the experimental results of performing MPSOIS and HPSOIS in 6D and 10D cases are basically same besides the result of MPSOIS in 10D case. Both of them do not show any change in search performance with the variation of the confidence coefficient w_3, i.e. their fitness values are always 1.0. Second, with the value of the confidence coefficient w_3 increases, the search performance of each proposed method

also increases. However, when w_3 approaches 1.0, the search performance of MPSOIWIS massively decrease. Third, for dealing with the $Schwefel$ problem, the search performance of all the proposed methods are not monotonous increasing. Especially, large oscillation of the search performance can be seen at $w_3 < 0$ case. Fourth, the search performance massively decrease with the increase of dimensions for handling the $Rastrigin$ problem by using MPSOIWIS.

According to the above search comparison and analysis, the effectiveness of information sharing and the search ability and performance of the proposed methods can be confirmed. It is clearly shown that the effect of cooperative and inhibitive roles with the variation of the confidence coefficient w_3. And an appropriate value of it is set case by case for enhancing the search ability and performance of the proposed methods.

4.3 Coefficient decision

As a new type approach of PMSO, how to determine the appropriate value of the confidence coefficient w_3 for efficiently solving the given optimization problems is an important task.

In order to confirm the most effectiveness of the proposed MPSOIS, MPSOIWIS, and HPSOIS, many computer experiments were carried out for observing the change of the obtained results in Section 4.2. Here, Figure 9 shows the curves of the average values obtained from each measurement curves shown in Figures 7 and 8. From the results shown in Figure 9, it can be seen that the curves of these fluctuations are single peaks in 6D case and 10D case, respectively.
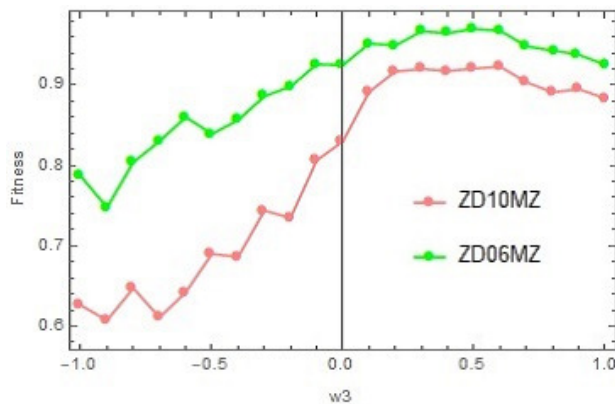

Figure 9. The curves of the average search results of each measurement value in 6D case and 10D case. ZD06MZ and ZD10MZ refer to the search results obtained in 6D case and 10D case, respectively.

The confidence coefficient w_3 corresponding to this peak value is found out to be approximately 0.5. Thus, to efficiently handle these given optimization problems by using

the basic search methods of PMSO, the value of the confidence coefficient w_3 should be set at 0.5.

With the change of the confidence coefficient w_3, the curve of the fitness in 10D case is generally lower than that of the fitness in 6D case. This is because of that the given optimization problems are complicated with dimensional increment.

4.4 Result comparison

To analyze and compare the obtained experimental results, furthermore, we compare these results that are before and after the introduction of the confidence term to the multiswarm search, and how much cooperative or inhibitive effect are achieved by performing the proposed methods.

Here, as the estimation indexes to the obtained experimental data, the maximum cooperative effect (MCE) and maximum inhibitive effect (MIE) are defined as follows:

$$MCE = \frac{f_{max} - f_{w_3=0}}{f_{max} - f_{min}} \qquad (10)$$

$$MIE = \frac{f_{w_3=0} - f_{min}}{f_{max} - f_{min}} \qquad (11)$$

where f_{max} and f_{min} are the obtained maximum value and minimum value of the obtained fitness, respectively. And $f_{w_3=0}$ is the obtained value of the used fitness at $w_3 = 0$ case, i.e. the search results of the existing search methods that are MPSO, MPSOIW and HPSO. Note that these maximum and minimum of the fitness values are not obtained at $w_3 = 1.0$ or $w_3 = -1.0$ cases.

Table 3. Search performance of the proposed methods for dealing with the given benchmark problems in 6D case

problem	method	$MCE(\%)$	$MIE(\%)$
	MPSOIS	–	–
$Sphere$	MPSOIWIS	0	100
	HPSOIS	–	–
	MPSOIS	–	–
$Schwefel$	MPSOIWIS	0	100
	HPSOIS	0	100
	MPSOIS	46.70	53.30
$Griewank$	MPSOIWIS	21.67	78.33
	HPSOIS	22.02	77.98
	MPSOIS	0	100
$Rastrigin$	MPSOIWIS	60.71	39.26
	HPSOIS	10.40	89.60

In Table 3, the symbol "–" means that since the values of

f_{max} and f_{min} are the same, the value of MCE and MIE can not be calculated.

Table 3 gives the calculating results of MCE and MIE in D6 case for dealing with the given benchmark problems. It is obvious that the performance effectiveness of the proposed methods have been increased by maximizing the potential search ability of the basic search methods of PMSO themselves. Regardless of the used methods, it is found that MCE and MIE greatly fluctuate depending on the difficulty level of the given benchmark problems, and both of them have not an asymmetrical relationship.

Table 4. Search performance of the proposed methods for dealing with the given benchmark problems in 10D case

problem	method	MCE(%)	MIE(%)
Sphere	MPSOIS	–	–
	MPSOIWIS	0.23	99.77
	HPSOIS	0	100
Schwefel	MPSOIS	0	100
	MPSOIWIS	25.22	74.78
	HPSOIS	11.74	88.26
Griewank	MPSOIS	6.95	93.05
	MPSOIWIS	14.03	85.97
	HPSOIS	24.17	75.83
Rastrigin	MPSOIS	9.82	90.18
	MPSOIWIS	71.44	28.56
	HPSOIS	66.23	33.77

Table 4 gives the calculating results of the MCE and MIE in 10D case for dealing with the given benchmark problems. By comparing both of the numerical results, we can confirm that the inhibitive effect in 10D case is higher than that in 6D case. Moreover, it is found that more the complicated problem is, the higher the search ability of the proposed methods is. And, this means that the search ability and performance of the proposed methods (MPSOIS, MPSOIWIS, and HPSOIS) are greatly improved than that of the existing ones (MPSO, MPSOIW, and HPSO).

5. CONCLUSION

How to effectively utilize the obtained search information and share it to adjust particles' velocities has become a significant research topic in the area of PMSO. Concurrently, this is useful way to handle complex optimization problems with low-cost and simplicity in computation.

For improving the search ability and performance of the elementary multiple particle swarm optimizers, in this paper we presented the special strategy of multi-swarm information sharing to PMSO. And based on the special strategy, we proposed the four basic search methods that are multiple particle swarm optimizers with information sharing (MPSOIS), multiple particle swarm optimizers with inertia weight with information sharing (MPSOIWIS), multiple canonical particle swarm optimizers with information sharing (MCPSOIS), and hybrid particle swarm optimizers with information sharing (HPSOIS), respectively. They are made up of the four basic search methods of PMSO. Especially, HPSOIS is a mixed method which has the search characteristics of the PSOIS, PSOIWIS, and CPSOIS. The existence of this search method gives the concept of a new configuration, and obtains better search results in our computer experiments.

Due to inspect the search ability and performance of the proposed methods, several computer experiments were carried out to deal with the given suite of benchmark problems. According to the obtained findings, we confirmed the better performance effect and search characteristics of these methods even for dealing with multimodal optimization problems. Among them, the search ability and performance of MPSOIS and HPSOIS are better than that of others in our experimental results.

For the sake of the reason, search characteristics of multiple particle swarm optimizers with information sharing plays an important role. Even more the obtained experimental results show that the proposed methods have better search ability and performance than those methods without the strategy of multi-swarm information sharing. And based on the obtained experimental results, the confidence coefficient w_3 should be set at 0.5 for efficiently dealing with the given optimization problems.

Owing to advantage of the excellent search capabilities of the proposed methods, MPSOIS and HPSOIS, furthermore, we are planning to develop a new type approach of intelligent PMSO (called IPMSO) by introducing some judgment strategies for dealing with data mining, pattern classification, system identification, multi-objective optimization problems as well as practical problems in the real-world.

REFERENCES

[1] Eberhart RC, Kennedy J. A new optimizer using particle swarm theory. Proceedings of the sixth International Symposium on Mi-

cro Machine and Human Science. 1995; 39-43. Nagoya, Japan. https://doi.org/10.1109/MHS.1995.494215

[2] Kennedy J, Eberhart RC. Particle swarm optimization. Proceedings of

the 1995 IEEE International Conference on Neural Networks. 1995; 1942-1948. Perth, Australia.

[3] Solis FJ, Wets RJ-B. Minimization by Random Search Techniques. Journal of Mathematics of Operations Research. 1981; 6(1): 19-30. Available from: `https://www.jstor.org/stable/3689263` (accessed 11 Dec. 2018).

[4] Spall JC. Stochastic Optimization, Handbook of Computational Statistics. J. Gentle et al. Eds. (Springer Heidelberg). 2004; 169-197.

[5] Bergh FVD, Engelbrecht AP. Training product unit neural networks using cooperative particle swarm optimisers. Proceedings of IEEE International Joint Conference on Neural Networks. 2001; 1: 126-131. Washington, D.C., USA. `https://doi.org/10.1109/IJCNN.2001.939004`

[6] Juang CF. A Hybrid of Genetic Algorithm and Particle Swarm Optimization for Recurrent Network Design. Journal of IEEE Transactions on Systems, Man and Cybernetics. 2004; Part B, 34(2): 997-1006. `https://doi.org/10.1109/TSMCB.2003.818557`

[7] Poli R, Kennedy J, Blackwell T. Particle swarm optimization – An overview. Journal of Swarm Intelligence. 2007; 1(1): 33-57. `https://doi.org/10.1007/s11721-007-0002-0`

[8] Poli R. Analysis of the Publications on the Applications of Particle Swarm Optimisation. Journal of Artificial Evolution and Applications. 2008; Issue 1(3): 10 pages. 2008. Hindawi Publishing Corp., NY, USA. `https://doi.org/10.1155/2008/685175`

[9] Ramadan RM, Abdel-Kader RF. Face Recognition Using Particle Swarm Optimization-Based Selected Features. International Journal of Signal Processing, Image Processing and Pattern Recognition. 2006; 2(2): 51-66.

[10] Zhang Y, Wang S, Ji G. A Comprehensive Survey on Particle Swarm Optimization Algorithm and Its Applications. International Journal of Mathematical Problems in Engineering. 2015; ID 931256, 38 pages. `https://doi.org/10.1155/2015/931256` (accessed 11 Dec. 2018).

[11] Zhang Y, Gong D, Ding Z. Handling multi-objective optimization problems with a multi-swarm cooperative particle swarm optimizer. Exper Systems with Applications. 2011; 38: 13933-13941. `https://doi.org/10.1016/j.eswa.2011.04.200`

[12] Fogel LJ, Owen AJ, Walsh MJ. On the Evolution of Artificial Intelligence Proceedings of the Fifth National Symposium on Human Factors in Electronics. 1964; 63-76. San Diego, CA, USA.

[13] Goldberg DE. Genetic Algorithm in Search Optimization and Machine Learning, Reading (Addison-Wesley). 1989.

[14] Holland JH. Adaptation in Natural and Artificial Systems. University of Michigan Press; 1975.

[15] Reyes-Sierra M, Coello CAC. Multi-Objective Particle Swarm Optimizers: A Survey of the State-of-the-Art. International Journal of Computational Intelligence Research. 2006; 2(3): 287-308.

[16] Chang JF, Chu SC, Roddick JF, et al. A parallel particle swarm optimization algorithm with communication strategies. Journal of Information Science and Engineering. 2005; 21: 809-818.

[17] El-Abd M, Kamel MS. A Taxonomy of Cooperative Particle Swarm Optimizers. International Journal of Computational Intelligence Research. 2008; 4(2): 137-144.

[18] Belal M, Ghazawi EI. Multi-swarm optimization in dynamic envirenments. Journal of Applications in Evolutionary Computing, G. R. Raidl, Eds. LNCS, (Springer-Verlay). 2004; 19-26.

[19] Blackwell T, Branke J. Paralled models for particle swarm optimization. International Journal on Intelligent Cooperative Information Systems. 2004; 4(1): 100-111.

[20] Li Y, Zhan Z, Lin S, et al. Competitive and cooperative particle swarm optimization with infomation sharing mechanism for global optimization problems. Journal of Information Sciences. 2014; 293(2015): 370-382. `https://doi.org/10.1016/j.ins.2014.09.030` (accessed 11 Dec. 2018).

[21] Niu B, Zhu Y, He X, et al. MCPSO: A multi-swarm cooperative particle swarm optimizer. Journal of Applied Mathematics and Computation. 2007; 185: 1050-1062. `https://doi.org/10.1016/j.amc.2006.07.026` (accessed 11 Dec. 2018).

[22] Jarbouia B, Cheikha M, Siarryb P, et al. Combinatorial particle swarm optimization (CPSO) for partitioned clustering problem. Journal Applied Mathematics and Computation. 2007; 192(15), Issue 2, 337-345. `https://doi.org/10.1016/j.amc.2007.03.010`

[23] Shi Y, Eberhart RC. Fuzzy Adaptive Particle Swarm Optimization. Proceedings of IEEE Congress on Evolutionary Computation. 2001; 101-106. Seoul, Korea. `https://doi.org/10.1109/CEC.2001.934377`

[24] Yin PY. Genetic particle swarm optimization for polygoral approximation of digital curves. Journal of Pattern Recognition and Image Analysis. 2006; 16(2): 223-233.

[25] Brits R, Engelbrecht AP, Bergh FVD. Niche Particle Swarm Optimization. *Technical Report*, Department of Computer Science, University of Pretoria. 2005.

[26] Valle YD, Venayagamoorthy GK, Mohagheghi S, et al. Particle Swarm Optimization: Basic Concepts, Variants and Applications in Power Systems. Journal of IEEE Transactions on Evolutionary Computation. 2008; 12(2): 171-195. `https://doi.org/10.1109/TEVC.2007.896686`

[27] Eberhart RC, Shi Y. Comparing inertia weights and constriction factors in particleswarm optimization. Proceedings of 2000 IEEE Congress on Evolutionary Computation. 2000; 1: 84-88. San Diego, CA, USA. `https://doi.org/10.1109/CEC.2000.870279`

[28] Shi Y, Eberhart RC. A modified particle swarm optimiser. Proceedings of the IEEE International Conference on Evolutionary Computation. 1998; 69-73. Anchorage, Alaska, USA. `https://doi.org/10.1109/ICEC.1998.699146`

[29] Clerc M, Kennedy J. The particle swarm-explosion, stability, and convergence in a multidimensional complex space. Journal of IEEE Transactions on Evolutionary Computation. 2000; 6(1): 58-73. `https://doi.org/10.1109/4235.985692`

[30] Clerc M. Particle Swarm Optimization. 2006; ISTE Ltd., London.

[31] Innocente MS, Sienz J. Particle Swarm Optimization with Inertia Weight and Constriction Factor. Proceedings of International Conference on Swarm Intelligence. 2011; 1-11. Cergy, France.

[32] Trelea IC. The particle swarm optimization algorithm: convergence analysis and parameter selection. Journal of Information Processing Letter. 2003; 85: 317-325. `https://doi.org/10.1016/S0020-0190(02)00447-7` (accessed 11 Dec. 2018).

[33] Sedighizadeh D, Mashian E. Particle Swarm Optimization Networks, Taxonomy and Application. International Journal of Computer Theory and Engineering. 2009; 1(5): 486-502. `https://doi.org/10.7763/IJCTE.2009.V1.80`

[34] Zhang H. A New Expansion of Cooperative Particle Swarm Optimization. Proceedings of the 17th International Conference on Neural Information Processing (ICONIP2010), Part I, LNCS 6443, Neural Information Processing – Theory and Algorithms. 2010; 593-600. Sydney, Australia.

[35] Zhang H. Multiple Particle Swarm Optimizers with Inetia Weight for Multi-objective Optimization. Proceedings of the International MultiConference of Enngineers and Computer Scientists 2012. 2012; 1: 19-26. Hong Kong, China.

[36] Sho H. Particle Multi-Swarm Optimization: A Proposal of Multiple Particle Swarm Optimizers with Information Sharing. Proceedings of 2017 IEEE 10th International Workshop on Computational Intelligence and Applications. 2017; 109-114. Hiroshima, Japan. `https://doi.org/10.1109/IWCIA.2017.8203570`

[37] Suganthan PN, Hansen N, Liang JJ, et al. Problem Definitions and Evaluation Criteria for the CEC 2005. Available from: `http://www.cil.pku.edu.cn/resources/psopaper/src/Optimization-Tech-Report-May-30-05.pdf` (accessed 11 Dec. 2018).